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CALCULATION OF TUBULAR RADIATORS OF THE AUTOMOBILE TYPE

By L. Richter

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CALCULATION OF TUBULAR RADIATORS OF THE AUTOMOBILE TYPE.*

By L. Richter.

The cooling capacity of automobile and airplane radiators has, hitherto, had to be determined experimentally for each particular type and, indeed, for every change in the dimensions and in the velocity of the air, since, in spite of valuable researches, no reliable general method of calculation was known.**

We propose to show how to calculate the cooling capacity of all radiators through which the air flows in separate streamlets, whether enclosed in actual tubes or not and whatever cross-sectional shape the tubes may have.*** The first part will give the fundamental principles for calculating velocity of the air in the tubes and the heat exchange by radiation, conduction and convection, and show, by examples, the agreement of the calculation with experiments.**** In the second part, the effect of the dimensions and conditions of operation on the heat exchange will be systematically investigated.

* From "Zeitschrift für angewandte Mathematik und Mechanik," August, 1925, pp. 293-313.

** Pülz, "Kühlung und Kühler für Flugmotoren," Berlin, 1920; Praetorius, "Die Kühlung leichter Verbrennungsmotoren," Berlin, 1921.

*** Pulz, above reference, p.73. According to this definition, the following investigation also includes the so-called ribbed or gilled radiators.

**** This section contains the essential portion of the writer's graduation dissertation at the Dresden Technical High School.

CALCULATION PRINCIPLES AND EXAMPLES.

1. Velocity of the air in the radiator.- The radiator is supposed to be subjected to an air stream of the same cross-section as the face of the radiator and flowing everywhere parallel to the axes of the air tubes. The angle between the air stream and the radiator face is assumed to differ but little from a right angle. If

F_{st} is the area of the radiator face in m^2,

F_e is the total free cross-section of the air intake,

F_1, F_2...F_n is the total free cross-section of the inside air passages at different distances from the radiator face and if

F_a is the total free cross-section of the air exit, then the air permeability of the radiator is represented by the ratios

$\varphi_e = F_e/F_{st}$, $\varphi_n = F_n/F_{st}$ and $\varphi_a = F_a/F_{st}$.

The radiator consists of n like tubes of the length l (in m) and of either constant or variable cross-section and is characterized by the interior cross-sectional area f and its circumference u or the equivalent diameter,

$$d = 4\ f/u \tag{1}$$

The latter is measured in mm, unless otherwise stated and is, for the circular cross-section, equal to the geometric diameter. For a square cross-section, d equals one side of the square.

The free air stream in front of the radiator has the rela-

tive velocity W_f. The air in the entrance cross-section has the mean relative velocity W_e. In the cross-sections F_1, F_2, ...F_n, F_a and behind the radiator, the air stream has the respective relative velocities W_1, W_2...W_n, W_a and W_h, all measured in m/s. The disturbance of the flow by the fan, engine and other obstructions, either in front of or behind the radiator, which are difficult to determine mathematically, was disregarded, as likewise any negative pressure behind the radiator due to the manner of installing.

The energy of the air stream in front of the radiator is partially lost during the entrance, passage through the radiator and exit and is partially retained in the emerging air stream. For 1 kg of air, we accordingly have*

$$\frac{W_f^2}{2\,g} = \zeta_e \frac{W_e^2}{2\,g} + \sum_{i=1}^{n} \beta_i\, W_i^2\, \frac{\Delta l}{d_i} + \zeta_a \frac{W_a^2}{2\,g} + \frac{W_h^2}{2\,g} \tag{2}$$

Furthermore, we have the continuity equations

$$\left.\begin{array}{ll} W_e = (W_1/F_e)W_1 = (\varphi_1/\varphi_e)W_1, & W_a = (F_1/F_a)W_1 = (\varphi_1/\varphi_a)W_1 \\ W_1 = (F_n/F_1)W_n \dots\dots & W_h = (F_1/F_{st})W_1 = \varphi_1\ W_1 \end{array}\right\} \tag{3}$$

from which we obtain

$$W_1 = \frac{W_f}{\sqrt{\zeta_e \frac{\varphi_1^2}{\varphi_e^2} + 2g \sum_{i=1}^{n} \beta_i\ \frac{\Delta l}{d_i}\left(\frac{F_1}{F_i}\right)^2 + \zeta_a \frac{\varphi_1^2}{\varphi_a^2} + \varphi_1^2}} \tag{4}$$

* Only approximate, because it is assumed that the static pressure in the free air stream before and immediately behind the radiator is the same. The later examples demonstrate the correctness of this assumption.

and also the values of W_2 and $W_3 \ldots W_n$.

For tubes of constant cross-section

$$F_e = F_1 = \ldots\ldots = F_n = F_a = F$$

$$\varphi_1 = \varphi_2 = \ldots\ldots = \varphi_n = \varphi_e = \varphi_a = \varphi$$ and therefore the passage velocity

$$W = \frac{W_f}{\sqrt{\zeta_e + 2g\,\beta\,l/d + \zeta_a + \varphi^2}} \tag{4a}$$

The coefficients ζ_e and ζ_a for a sudden contraction or expansion (Hütte I, 24th edition, pp. 376 and 380) and φ^2 for the energy of the air behind the radiator, as likewise the sum $\zeta_e + \zeta_a + \varphi^2$, are combined in Fig. 1.

We have, according to Fritzsche,*

$$\beta = 6.02\ d^{-0.269}\ (\gamma\ W)^{-0.148} \tag{5}$$

in which γ denotes the mean specific gravity of the air in the radiator in kg/m . By introducing this and $g = 9.81$ into formula (4a) we obtain

$$W = \frac{W_f}{\sqrt{\zeta_e + \zeta_a + \varphi^2 + 118.1\ (\gamma\ W)^{-0.148}\ d^{-1.269}\ l}} \tag{4b}$$

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* Hütte I, 24th edition, p.536. According to this, we have, for the pressure loss in meters of the fluid column, $h = \beta\ W^2\ l/d$ with d in mm, l in meters and W in m/s, while, for smooth tubes, we would have $h = \lambda \frac{v^2}{2g}\ l/D$ with D in cm, l in m, v in cm/s, g in cm/s² and $\lambda = A\left(\frac{\nu}{vD}\right)^n$ with the kinetic viscosity ν in cm²/s.

For like tube dimensions and pressure losses, we would thus obtain $\lambda = 0.01962\ \beta$.

According to	Blasius	Ombeck	Fromm
A =	0.316	0.242	0.399
n =	0.25	0.224	0.27

(Cont. on next page)

The specific gravity of the entering air may be substituted for γ without serious error. In the first approximation, W may be substituted for W_f and the value of W be calculated from formula (4), (4a) or (4b). By putting the value thus obtained under the radical again, we obtain the value of W with sufficient accuracy.

In Table I, according to foreign experiments,* the values for the velocity W in the radiator tubes, as obtained from the velocity of the air behind the radiator and the continuity equation $W = W_h : \varphi$, are compared with the values obtained from formula (4) and show a good agreement with the experiments. The greater deviation in line 19 (16%) is probably due to the transition from vortical to laminar flow and in line 22 (19.6%) to an experimental error.

(Continuation of footnote from p.4.)

For comparing the different formulas, λ or $0.01963\,\beta$ is plotted in Fig. 2 for a tube of 6 mm diameter at an air pressure of 735.5 mm Hg, 30°C, specific gravity $\gamma = 1.13$ kg/m³ and a kinetic viscosity $\nu = 0.165$ for air velocities between 2 and 50 m/s. Velocities of less than 5 m/s, however, lie below the critical velocity.

The diagram demonstrates that the loss by friction may be characterized, according to Fritzsche's formula, as that of a slightly rough tube, whereby the approximation to a quadratic function of the pressure loss is characteristic of the velocity, as was also found in Fromm's experiments ("Zeitschrift für angewandte Mathematik und Mechanik," 1923, p.339). This was also (together with the convenience of the formula and the good agreement of the values obtained by its use for the velocity in the tubes with the experimental results) the reason why it was retained in spite of theoretical deficiency (See Fromm, above reference, p.343), since, moreover, the "roughness" could not yet be determined numerically.

* Von Doblhoff, "Untersuchung von Automobilkühlern" in "Mitteilungen über Forschungsarbeiten," No.93, a report on radiator experiments at the Vienna Engine Works.

The derived data, in fact, apply accurately only to a vortical flow. Fig. 3 shows, for air at 1 atm. and 20°C in tubes of different diameters, the critical velocities (Hütte I, 24th edition, p.536), below which we may expect deviations from the calculated values, although on account of the shortness of the tubes, the vortices in the inflowing air cannot yet come to rest. The velocities in automobile radiators probably lie, however, mostly in the region of the vortical flow, so that the given equations may be used without hesitation.

2. Heat transference by radiation.- How the order of magnitude of the ratio of the radiated heat to the total amount of heat transferred can be determined, will be shown with a Daimler V radiator, 200 mm deep,* which will also be used further as an example.

From a surface element d F, there will be transferred, by radiation per unit of time, the heat

$$d\,Q_{st} = c\,[\,(T_W/100)^4 - (T_0/100)^4]\;d\,F_1 \qquad (6)$$

$$1/c = 1/C_1 + (1/C_2 - 1/C)\,\frac{\Sigma F_1}{F_2} \qquad (7)$$

wherein, according to Nusselt,**

* Vienna Engine Works (See Pulz, previous reference, p.44, and "Motorwagen," XXIII/7, 10.
** Nusselt, "Der Wärmeübergang in der Gasmaschine" in "Mitteilungen über Forschungsarbeiten," No. 264, p. 7.

C_1 is the coefficient of radiation of the heat-giving body,

C_2 is the coefficient of radiation of the surrounding medium,

C is 4.7, the coefficient of radiation of the absolutely black body,

ΣF_1 is the radiating surface,

F_2 is the surface receiving the radiations.

T_W is the absolute temperature of the radiator wall, dependent on the position of the surface element, and

T_O is the absolute temperature of the surrounding medium.

T_W is, at most, equal to the temperature of the circumfluent water. For the approximate calculation of the heat radiated from the lateral, front and inner surfaces of the tubes, on the assumption of a linear temperature fall in the direction of the water flow, we put

$$T_W = T_{WE} - \theta y \tag{8}$$

T_{WE} is the entrance temperature of the water,

$$\theta = \frac{T_{WE} - T_{WA}}{h} \tag{9}$$

the temperature drop, T_{WA} the exit temperature of the water, h the height of the radiator or (in radiators in which the water does not flow vertically) the distance between the entrance and exit and y the vertical distance between the given surface element and the water entrance.

If we introduce equation (8) into equation (6) and integrate between 0 and h, we then obtain, for radiators with rec-

tangular front and uniform depth, the total heat radiated from surfaces with variable temperature

$$Q_{st} = c/100^4 \Sigma F_1 [T_{WE}^4 - 2\ T_{WE}^3 h\theta + 2\ T_{WE}\theta_2 h^2 - T_{WE}\theta^3 h^3 + \frac{1}{5}\theta^4 h^4 - T_0^4] \quad (10)$$

If we imagine the radiator, surrounded by a relatively large sphere, then some parallel of the spherical surface F_2 will closely correspond to each portion of the outer surface of the radiator. On the other hand, however, the radiation from the tubes, through their own walls, is limited.

To the surface element $d\ f_1 = u\ d\ x$ (Fig. 4), there corresponds only a fully capable radiation element $d\ f_{n\,1} = \sin\alpha\ d\ f_1$, which radiates through an end section on to a parallel element of the sphere in such manner that the maximum angle α is given by the distance d of the portion of the surface from the opposite surface and the distance x of the surface element from the front wall of the radiator:

$$\sin\alpha = \frac{\alpha}{\sqrt{x^2 + d^2}} \quad (11)$$

By integration over the length of the tube, whereby the value, corresponding to the two openings of the tube, is to be introduced double, we obtain, as the equivalent radiating surface for the core consisting of n tubes,

$$F_{n\,1} = n\ f_n = n\ u\ \text{arc tan}\ l/d \quad (12)$$

On the radiator "Daimler V, 200 mm deep," the upper surface,

0.485 × 0.2 = 0.097 m², at a constant absolute wall temperature of 349.4°, radiates, according to equation (10) with $C_1 = 1.5$, $C_2 = 4.7$ (the "surrounding medium" being considered as absolutely black) and at a temperature of 293° absolute, 11.3 kg-cal/h, while the lower surface radiates 8.3 kg-cal/h at a constant absolute wall temperature of 337.9°.

The equivalent tubular surface was calculated from equation (12) to be 1.17 m², while the heat radiated by it was calculated as for the lateral surfaces of 2 × 0.28 × 0.2 = 0.112 and for the radiating portion of the frontal surfaces $2\,(1 - \varphi_e)\,F_{st} = 0.079$, from equation (10) with

$$h = 0.280 \text{ m} \quad \text{and} \quad \theta = \frac{349.4 - 337.9}{0.280} = 41$$

to amount in all to 135 kg-cal/st^{-1} .

The total heat radiated therefore amounts to

$$11.1 + 8.3 + 135.1 = 154.5 \text{ kg-cal/h.}$$

With the above limiting value $C_1 = 4.7$, the radiated heat is 478.5 kg-cal/st^{-1}, which is about 1.3% of the amount transferred by conduction and convection.

3. Heat transmission by conduction and convection.- The heat Q_N transmitted by conduction and convection through the wall between the water and the air can be approximated by

$$\left.\begin{aligned} Q_N &= \frac{t_{WE} - t_{LE}}{\frac{1}{k\,F} + \frac{1}{2\,M} + \frac{1}{2\,m}} = K_n\,M\,(t_{WE} - t_{LE}) \\ \text{with}\quad & \frac{\frac{k\,F}{M}}{1 + \frac{k\,F}{2\,M} + \frac{k\,F}{2\,m}} \end{aligned}\right\} \qquad (13)$$

when both fluids are heated by a temperature difference, which is small in comparison with the difference between the mean temperatures (mean between entrance and exit temperatures) on both sides of the separating wall.* Here t_{WE} and t_{LE} are the entrance temperatures of the water and air in degrees centigrade; M and m, the water equivalents in kg-cal/h/°C of the quantities of air and water flowing through per unit of time; F, the radiator surface in m²; and k, the coefficient of heat conductivity in kg-cal/h/m²/degree.

For great temperature changes in the fluids, equation (13) must be replaced by the equation developed by Nusselt for heat transmission in the cross-current. With our previous symbols

$$Q_K = M\,(t_{WE} - t_{LE} - \frac{1}{l}\int_0^l t_{y=h}\,d\,x \qquad (14)$$

with

$$t_{y=h} = (t_{WE} - t_{LE})\,e^{-\frac{kF}{M}}\left[1 + \int_0^{\frac{k^2F^2}{m\,M}Xl} e^{-\frac{M}{kF}Z} \frac{iJ_1\,(2i\sqrt{Z})}{\sqrt{Z}}\,dz\right] \qquad (15)$$

Here e is the basis of the natural logarithms and J_1 is the Bessel function of the first kind and order. By introducing

* See footnote next page.

equation (15) into equation (14), we obtain

$$Q_K = M(t_{WE} - t_{LE}) \left[1 - \frac{e^{-\frac{kF}{M}}}{l} \int_0^l \left(1 + \int_0^{\frac{k^2F^2}{M\ m} x/l} e^{-\frac{M}{kF}Z} \frac{iJ_1(2i\sqrt{Z})}{\sqrt{Z}} dz\right) dx\right] \qquad (16)$$

Table II.

Correction coefficient $q = K_K/K_N$ for taking the cross-current into account.

kF/M	KF/m=	0.0	0.5	1.0	2.0	5.0	10.0	15.0
0.0		1.0	0.984	0.948	0.865	0.695	0.600	0.567
0.2		0.997	0.986	0.960	0.903	–	–	–
0.5	q =	0.984	0.979	0.968	0.923	0.690	–	–
1.0		0.948	0.940	0.938	0.916	0.744	–	–
1.5		0.908	0.907	0.902	0.891	0.776	–	–
2.0		0.865	0.863	0.862	0.861	0.838	0.732	0.644

From Table II for

$$0 \leqq \frac{kF}{m} \leqq 15 \text{ and } 0 \leqq \frac{kF}{M} \leqq 2$$

the value of the proportionality factor

$$q = \frac{Q_K}{Q_N} = \frac{K_K}{K_N} \qquad (16a)$$

is to be taken, so that also, according to equation (14) the calculation is simplified.

In the coefficient of heat transmission

$$k = \frac{1}{\frac{1}{\alpha_L} + \frac{\alpha F_L}{\lambda F_r} + \frac{1}{\alpha_W}\frac{F_L}{F_W}} \qquad (17)$$

(*Footnote from p. 10.)
Nusselt, "Der Wärmeübergang im Kreuzstrom," Z.d.V.d.I. 1911, p.2021. The approximation equation (35) applies to the case when only the temperature of the air, but not of the water in the tubes, increases or decreases linearly in the direction of flow.

in which α_L and α_W denote the coefficients of heat transmission on the wall for air and water; F_L, the radiator surface in contact with the air; F_W, the radiator surface in contact with the water; and F_r, a reduced radiator surface determined by the geometrical shape of the wall. α/λ (α the thickness of the wall and λ the coefficient of heat conduction of the wall), under the existing relations with respect to $1/\alpha_L$ and $1/\alpha_W$, is very small and may be disregarded.

In the case of iron, that is, with $\lambda = 60$ and with a wall thickness of 0.1 - 0.3 mm

$\alpha/\lambda = 0.00166 \times 10^{-3}$ to 0.005×10^{-3}, while

$1/\alpha_L$ lies between 1 and 6×10^{-3} and

$1/\alpha_W$ lies between 2×10^{-3} and 0.5×10^{-3}.

According to Nusselt,* the mean coefficient of heat transmission for gases in the tubes, in a vertical flow, at the velocity W, is

$$\alpha_{Lm} = 22.60(\lambda_m/D)\ (D/l)^{0.054}\left(\frac{DW\lambda_m C_{pm}}{\lambda_m}\right)^{0.786} \tag{18}$$

in kg-cal/m²/h/°C. Herein the equivalent diameter D is to be given in meters.

If we assemble the like members and if we fix the specific heat for 1 kg of air between -30°C and 70°C under constant pressure at

$$C_{pm} = 0.240 \text{ kg-cal/kg/}^\circ\text{C}$$

* Nusselt, "Der Wärmeübergang im Rohr," Z.d.V.d.I. 1917, p.686. Hütte, 24th edition, I, p.459.

whereby the error amounts to plus or minus one to two thousandths, then

$$\alpha_{Lm} = 22.23\ \lambda_m^{0.214}\ d^{-0.160}\ l^{-0.054}(W\ \lambda_m)^{0.786} \qquad (18a)$$

wherein d is to be put in millimeters. Theoretically, the values for the coefficient of heat conduction λ_m and the specific gravity λ_m should be taken at the mean temperature of the air and walls. It is sufficiently accurate, however, for the first approximation and often also for the final value, to take it for the temperature $\frac{t_{LE} + t_{WF}}{2}$.

The heat transmission on the lateral surfaces* can be calculated, at an atmospheric pressure of 752 mm Hg, a wall temperature of 50°C and an air temperature of 20°C, from

$$\alpha_L = 6.14\ W_f^{0.78} + 4.60\ e^{0.6\ W_f}\ \text{kg-cal/m}^2\text{/h/°C} \qquad (19)$$

wherein W_f is the wind velocity in m/s and e is the basis of the natural logarithms. Any change in this coefficient of heat transmission, due to changes in temperature, may be disregarded in the present calculation, since the corresponding surfaces are relatively small.

The formula

$$\alpha_L = 0.0670\ (\lambda_m/D)\left(1273 + \frac{D\ w_0\ \rho_m}{\eta_m}\right)^{0.716} \qquad (20)$$

for cooling a cylinder by means of an air stream perpendicular to the axis,** is to be used for the heat transmission on the

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**Nusselt, "Die Kühlung eines Zylinders durch einen senkrecht zur Achse strömenden Luftstrom," "Gesundheits-Ing." 1922, p.97.

*Nusselt, "Die Kühlung einer ebenen Wand durch einen Luftstrom" in "Gesundh. Ing." 1922, p. 641.

front surface of a radiator having tubes with rectangular cross-sections at their ends because the flow around the ends of the thin partitions between the air passages has a certain similarity with the flow around a cylinder having a diameter equal to the thickness of the partitions. Errors in the value of α_L for the sides and front surface have, moreover, but little effect on the final result since these surfaces are small in comparison with the total inside surface of the tubes.

In equation (20), D is expressed in meters. The mean density $\rho_m = \gamma_m/9.81$ and the viscosity of the air is

$$\eta = \frac{2.41 \times 10^{-6}}{1 + \frac{117}{T}} \sqrt{\frac{T}{273}} \text{ kg/s/m} \qquad (21)$$

With regard to the differences in the velocity of the air before and behind the radiator, it is advisable, in calculating the coefficient of heat transmission according to equation (20), to use the velocity W in the tubes. From the thus-calculated heat-transmission coefficients α_L and the corresponding areas, it is advisable to obtain, before introduction in k, the mean value

$$\alpha_{Lm} = \frac{\Sigma F \alpha_m}{\Sigma F}$$

The effect of the portion of the cooling surfaces not in contact with the water is to be disregarded. It is evident from Fig. 5, a section through adjoining tubes parallel to the front surface, that the heat transmission can be compared to the case

of a plate with uniform temperatures t_W in the end sections. For a plate thickness a, which is small in comparison with d, we obtain from

$$\lambda a \frac{d^2 t}{dx^2} - 2\,\alpha\,(t - t_L) = 0 \tag{22}$$

in which t denotes the temperature in the given cross-section, t_L the temperature of the air, x the distance from a wall, α_L the coefficient of heat transmission and λ the coefficient of heat conduction,

$$\frac{t-t_L}{t_W-t_L} = \frac{1}{2\sin d_2\sqrt{\frac{2\alpha}{\lambda a}}}\left[\left(1-e^{-d_2\sqrt{\frac{2\alpha}{\lambda a}}}\right)e^{x\sqrt{\frac{2\alpha}{\lambda a}}}-\left(1-e^{d_2\sqrt{\frac{2\alpha}{\lambda a}}}\right)e^{-x\sqrt{\frac{2\alpha}{\lambda a}}}\right]$$

and, for the mean value of the wall temperature t_m,

$$\left.\begin{aligned} \frac{t_m - t_L}{t_W - t_L} &= \frac{2\,(\cos\varphi - 1)}{\varphi\,\sin\varphi} \\ \text{with}\qquad \varphi &= d_2\sqrt{\frac{2\,\alpha_L}{\lambda}} \end{aligned}\right\} \tag{24}$$

The value $\theta = \dfrac{t_m - t_L}{t_W - t_L}$ can be used as a reduction factor for the unwet cooling surface and is to be taken from Fig. 6.

The widths of the sides of the tubes lie between 4 and 10 mm, the wall thicknesses between 0.1 and 0.3 mm and the cooling effect of the intermediate cooling surfaces is only slightly less at values for $\alpha_L < 150$ and the given widths of the immediate cooling surfaces (in contact with water). This differs from Pülz on p.83 of the reference already given.

Since Schiller ("Ueber den Stromungswiderstand von Rohren," Z.f.angew. Math. und Mech." 1923, p. 9) shows that laws of flow for cylindrical tubes, after the introduction of the equivalent diameter $d = \frac{4f}{u}$, also hold true for square tubes with very different side widths and the laws of heat transmission are intimately related to those of flow resistance, it will not seem too bold to use the formulas of Sonnecken ("Mitt. über Forschungsarbeiten," Nos. 108-109) for the heat transmission of water in rough cylindrical tubes

$$\alpha_W = 735 \frac{w^{0.7}}{D^{0.3}} (1 + 0.014\ t_i) \tag{25}$$

in which D is the tube diameter in m, w the mean water velocity in m/s, and t_i the wall temperature, also for heat transmission in radiators, where the water passages are long narrow rectangular tubes of the length l and width g. In this case $D = \frac{4\ l\ g}{2(l+g)}$ and, for $l \gg g$,

$$D \sim 2\ g \tag{26}$$

However, in radiators whose air tubes are surrounded on all sides by water, the coefficient of heat transmission for water can be only estimated or determined from experiments, e.g., those of Doblhoff, in the following manner. If t_W and t_L are the mean water and air temperatures on both sides of the wall, whose mean temperatures, on the water and air sides, are respectively, t_1 and t_2, then the transmitted heat, known from

the experiments, is

$$Q=\alpha_W F\ (t_W-t_1)=(\lambda/d)F_{red}(t_1-t_2)=\alpha_L F_L(t_2-t_L)=k\ F_L(t_W-t_L) \qquad (26)$$

In our cases, λ/d is always very great and $t_1 - t_2$ consequently very small, so that t_1 approximately equals t_2 and either one may be considered as equal to t. The wall surface F_L, in contact with the air, is approximately equal to the sum of F_W (in contact with the water) and F_N (not in contact with the water). With calculated α_L we find

$$t = t_L + \frac{Q}{\alpha_L\ F}.$$

and the desired coefficient for the transmission from the water to the tubes

$$\alpha_W = \frac{Q}{F_W\left(t_W - t_L - \dfrac{Q}{\alpha_L\ F_L}\right)} \qquad (27)$$

For $F_W = F_L$

$$\alpha_W = \frac{Q}{F_L(t_W - t_L) - Q/\alpha_L} \qquad (27a)$$

The arithmetical mean between the inflow and outflow temperatures are to be substituted for the temperatures t_W and t_L. In what follows, the values of α_W (from equation (27)) is used for comparison with the value calculated from equation (25).

The water equivalent of the air passing through the tubes

$$m' = G_L\ c_p = 3600 \times W_i\ F_i\ \gamma_m\ c_p \qquad (28)$$

is to be multiplied by $\dfrac{n\ u\ l + 2(b + h)\ l\ (W_f/W)}{n\ u\ l}$

in order to obtain the total value including the share of the outer surface.

$$m = 3600\ W_i\ F_i\ \frac{n\ u + 2(b + h)\ (W_f/W)}{n\ u} \qquad (28a)$$

The mean value for inflow and outflow, or approximately at first the value of the inflow temperature. After a more accurate calculation of the total transmitted heat Q and therewith the outflow temperature, γ_m and Q are to be corrected.

4. Examples and comparison with experiments.

a) Daimler V, 200 mm deep.- Table I gives the dimensions of the radiator.

W_f = 38.6 m/s, velocity of air in front of radiator,

t_{LE} = 25.8°C, temperature of air in front of radiator,

G = 4030 kg/h, quantity of water flowing through radiator,

t_{WE} = 76.43°C, temperature of inflowing water,

γ_1 = 1.16 kg/m³, specific gravity of air in front of radiator.

From equation (4b) we obtain

$$W = 25.1 \text{ m/sec.}$$

as the velocity of the air in the radiator. The temperature for the heat transmission to the air in the tube is approximately put at $\dfrac{t_{LE} + t_{WE}}{2} = 51.1°C$ and the corresponding specific gravity accordingly at

$$\gamma_m = 1.16 \frac{273 + 25.3}{273 + 51.1} = 1.07 \quad \text{and} \quad \gamma_m W = 26.9.$$

With $\lambda_m^{0.214} = 0.448$, $l^{-0.054} = 1.091$, $d^{0.160} = 0.759$ and $(\gamma_m W)^{0.786} = 13.26$, we obtain, according to equation (18a), for the inner surface

$$\alpha_{L1} = 109.5 \text{ kg-cal/m}^2\text{/h/}^\circ\text{C}$$

The inner surface, cooled by direct contact with the air, is 6.82 m², while the same area indirectly cooled and multiplied by the reduction factor $\theta = 0.983$, for which $\alpha_L = 110$, $l = 100$, hence $\frac{\alpha_L}{\lambda} = 1.1$, is 6.71 m², thus giving a total area of 13.53 m². For the outer surface of 0.307 m² at $W_s = 38.6$, $\alpha_{L2} = 105.5$. For both frontal surfaces, with web 2 mm and 0.86 mm wide, and 0.055 m² and 0.0238 m² total area, $\alpha_{L3} = 310$ and 480 at $W = 25.6$. The mean value for $\alpha_L = \frac{\Sigma \alpha_L F}{\Sigma F}$ is 109.9. The water flows through 64 rectangular compartments of about 1.6 × 195 mm (with a uniform diameter 2 × 1.6 = 3.2 mm) in a total cross-section of 2 d m² with 0.56 m/s. Approximately $t_i = \frac{\alpha_L\, t_{LE} + \alpha_W\, t_{WE}}{\alpha_L + \alpha_W}$, so that, with $\alpha_W = 1100$ approximately $t_i = 71.5^\circ C$.

From equation (25) we obtain $\alpha_W = 1120$. According to equation (17), the coefficient of heat passage $k = 92.4$ and $kF = 1404$ and the water equivalent of the passing water $M = 4030$ kg-cal/h/°C. For air, according to equation (28a), $m = 2510$, $\frac{kF}{M} = 0.321$, $\frac{kF}{m} = 0.516$ and approximately (from

equation (13)) $K_n = 0.226$ or, more accurately, for a cross-current by multiplication by $q = 0.99$, $K_q = 0.224$ and (from equation (16)) the total transmitted heat $Q_k = 45800$ kg-cal/h, from which follow the air and water exit temperatures

$$t_{LA} = t_{LE} + Q_k/m = 44.1^\circ C \quad \text{and} \quad t_{WA} = t_{WE} - Q_k/M = 65$$

Generally this approximation suffices. More accurate values can be obtained by a recalculation in the following manner, using the numbers obtained in the approximate calculation.

The mean air temperature $t_{LM} = \dfrac{t_{LE} + t_{LA}}{2}$

The mean water " $t_{WM} = \dfrac{t_{WE} + t_{WA}}{2}$

The mean wall " $t_i = 67.5$

from which we obtain the mean temperature $\dfrac{35 + 67.5}{2} = 51.2^\circ C$ (above approximately 51.1) decisive for the calculation of the coefficient of heat transmission, $\gamma_m = 1.07$ and $\alpha_L = 109.9$ (as above), $\alpha_W = 4120 \times 0.136 \times 1.946 = 1090$, $k = 92$, $kF = 1292$ and $M = 4030$ (as above). On the contrary, we now obtain for the air in the tube $\gamma_m = 1.125$ and accordingly $m = 2430$, $\frac{kF}{M} = 0.321$, $\frac{kF}{m} = 0.532$, $K_n = 0.225$, $K_k = 0.223$ and the total transmitted heat $Q_k = 45500$ kg-cal/h. In the experiment, the temperature of the outflowing water was $64.9^\circ C$, which gives a total transmitted heat

$$Q = (76.43 - 64.9) \times 4030 = 46500$$

which differs by only 2.1% (radiation and error) from the calculated result.

As determined, according to equation (27), from calculated value of α_L, as likewise from the transmitted heat Q and the mean water and air temperature, $\alpha_W = 1140$ and accordingly agrees well with the result calculated from equation (25).

b) Dr. Zimmerman's radiator (Doblhoff's experiments).- The calculation corresponds perfectly to example a), excepting that the law for the transmission of the heat from the water to the round tubes has to be derived, for the lack of other sources, from Doblhoff's experiments according to equation (27). These experiments did not render it possible to establish the law in the sense of Nusselt's ("Das Grundgesetz des Wärmeüberganges" in "Gesundh, Ing." 1915, p.477), but the equation

$$\alpha_W = 90\ (2.5 + w)^{0.3}(1 + 0.008\ t) \qquad (29)$$

can generally be used, in which W is the velocity of the water in the unconstricted cross-section and t the mean between the temperature of the water and the wall. These can be accurately ascertained for tube diameters of 5-8 mm, usually employed in radiators, and for a temperature range of 40-90°C. In their order of magnitude, these coefficients of heat transmission agree with the values of equation (25).

TABLE I.

Experimental and Calculated Data.

No.	1	2	3	4	5	6
1	V	Source	-	Reports on radiator experiments by the Vienna Engine Works		
2	V	Experimental number of the source	-	7	7	7
3	V	Source and type of radiator	-	Daimler V 200 mm deep		
4	V	Outside dimensions of radiator, h × b × l	mm	280 × 485 × 200		
5	V	Frontal area b × h	m^2	0.1358		
6	V	Number, cross-section and length of tubes	mm	3015 × 5.65 × 5.65 × 200		
7	V	Thickness of wall	mm	0.17		
8	R	Equivalent diameter d	mm	5.65		
9	V	Total free cross-section of air intake F_e	m^2	0.0965		
10	V	Total free cross-section of inner air passages F_i	m^2	0.0965		
11	V	Total free cross-section of air outlet F_a	m^2	0.0965		
12	R	Air permeability at intake $\varphi_e = \frac{F_e}{F_{st}}$	-	0.711		
13	R	Air permeability within $\varphi_i = \frac{F_i}{F_{st}}$	-	0.711		
14	R	Air permeability at outlet $\varphi_a = \frac{F_a}{F_{st}}$	-	0.711		

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	4	5	6
15	V	Inner surface of tubes, in contact with water	m^2		6.84	
16	V	Inner surface of tubes, not in contact with water	m^2		6.80	
17	V	Surface of jacket	m^2		0.307	
18	V	Broad strips of frontal surface - width:area	mm/m^2		2/0.055	
19	V	Narrow strips of frontal surface - width:area	mm/m^2		0.86/0.024	
20	V	Total cooling surface (not reduced)	m^2		14.03	
21	V	Free cross-sections of water passages (maximum)	mm		64 × 1.6 × 190	
22	R	Equivalent diameter of water passages d_W	mm		3.2	
23						
24	V	Barometer reading	mm Hg	745.3	745.3	745.3
25	V	Temperature of inflowing air t_{LE}	°C	25.8	25.5	23.5
26	V	Temperature of inflowing water t_{WE}	°C	76.4	76.7	76.4
27	V	Difference in inflow temperature $t_{HC} - t_{LE}$	°C	50.6	51.2	53.1
28	R	$\frac{1}{2}(t_{WE} + t_{LE})$	°C	51.1	51.1	49.4
29	V	Density of inflowing air γ	kg/m^3	1.160	1.160	1.175
30						
31	V	Air velocity before radiator W_f	m/s	38.6	32.6	27

TABLE I (Contd.)
Experimental and Calculated Data.

No.	1	2	3	4	5	6
32	V	Air velocity behind radiator W_f	m/s	17.5	14.9	11.9
33	V	Mean air velocity in the tubes W_V	m/s	24.6	21.0	16.8
34	R	Mean air velocity in the tubes, calculated from eq. 3, 3a and 3b W_R	m/s	25.1	21.0	17.2
35	R	$100 \frac{W_R}{W_V}$	-	102.0	100.0	102.5
36	R	Coefficient of heat transmission for air, inner surface α_{L1}	kg-cal/m²/h/gr	109.5	95.2	81.2
37	R	Reduction factor for unwet inner surface ε	-	0.990	0.992	0.993
38	R	Area of reduced inner surface	m²	13.52	13.54	13.55
39	R	Coefficient of heat transmission, jacket surface α_{L2}	kg-cal/m²/h/gr	105.5	92.5	80.0
40	R	Coefficient of heat transmission, for broad strips of frontal surface α_{L3}	kg-cal/m²/h/gr	310	281	258
41	R	Coefficient of heat transmission, for narrow strips of frontal surface α_{L4}	kg-cal/m²/h/gr	480	415	385
42	R	Mean coefficient of heat transmission for air $\alpha_L = \frac{\Sigma\alpha F}{\Sigma F}$	kg-cal/m²/h/gr	109.9	95.8	82.3
43	R	Mean wall temperature t_i	°C	67.5	72.7	72.7

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	4	5	6
44	R	Mean temperature of marginal water layer $\frac{t_i+t_W}{2}$	°C	-	-	-
45	R	Mean coefficient of heat transmission for water from eq. 25 and 29 α_H	kg-cal/m²/h/gr	1090	1140	1140
46	V-R	Ditto according to eq. 27 from experiments α_{WV}	kg-cal/m²/h/gr	1140	987	1125
47	R	Mean coefficient of heat transmission k	kg-cal/m²/h/gr	92.0	82.3	72.2
48	R	k F	-	1292	1154	1022
49	R	Water equivalent of the water M	kg/h	4030	4010	4010
50	R	Density of air at $\frac{1}{2}(t_{LE}+t_{WE})$	kg/m³	1.070	1.068	1.078
51	R	Mean density of air at $\frac{1}{2}(t_{LE}+t_{LA})$	kg/m³	1.125	1.128	1.140
52	V.	Velocity of water W_W	mm/s	57.0	56.2	56.2
53	R	Water equivalent of the air m	kg/h	2430	2042	1692
54	R	$\frac{kF}{M}$		0.321	0.288	0.255
55	R	$\frac{kF}{m}$		0.532	0.564	0.604
56	R	K_n		0.225	0.202	0.1785
57	R	K_K		0.223	0.201	0.1773

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2		3	4	5	6
58	R	Total heat, Q_K	transmitted calculated	kg-cal/h	45500	41200	37700
59	V	Total heat, Q_V	transmitted by experiment	kg-cal/h	46500	41500	37600
60	R	$100\ \frac{Q_K}{Q_V}$			97.9	99.1	100.3

V stands for "Versuch" (experiment),

R stands for "Rechnung" (calculation).

TABLE I (Cont.
Experimental and Calculated Data.

No.	1	2	3	7	8	9
1	V	Source	-	Reports on radiator experiments by the Vienna Engine Works		
2	V	Experimental number of the source	-	8	8	8
3	V	Source and type of radiator	-	Daimler V 300 mm deep		
4	V	Outside dimensions of radiator h x b x l	mm	283 x 480 x 300		
5	V	Frontal area b x h	m^2	0.1358		
6	V	Number, cross-section and length of tubes	mm	2970 x 5.6 x 5.6 x 300		
7	V	Thickness of wall	mm	0.2		
8	R	Equivalent diameter d	mm	5.6		
9	V	Total free cross-section of air intake F_e	m^2	0.0933		
10	V	Total free cross-section of inner air passages F_i	m^2	0.0933		
11	V	Total free cross-section of air outlet F_a	m^2	0.0933		
12	R	Air permeability at intake $\varphi_e = \frac{F_e}{F_{st}}$	-	0.686		
13	R	Air permeability within $\varphi_i = \frac{F_i}{F_{st}}$	-	0.686		
14	R	Air permeability at outlet $\varphi_a = \frac{F_a}{F_{st}}$	-	0.686		

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	7	8	9
15	V	Inner surface of tubes in contact with water	m^2		10.08	
16	V	Inner surface of tubes, not in contact with water	m^2		10.00	
17	V	Surface of jacket	m^2		0.458	
18	V	Broad strips of frontal surface - width:area	mm/m^2		2/0.0595	
19	V	Narrow strips of frontal surface - width:area	mm/m^2		1/0.026	
20	V	Total cooling surface (not reduced)	m^2		20.6	
21	V	Free cross-sections of water passages (maximum)	mm		64 x 1.6 x 290	
22	R	Equivalent diameter of water passages d_W	mm		3.2	
23						
24	V	Barometer reading	mm Hg	743.6	743.6	743
25	V	Temperature of inflowing air t_{LE}	^oC	25.3	25.3	20.4
26	V	Temperature of inflowing water t_{WE}	^oC	73.2	74.9	73.0
27	V	Difference in inflow temperature $t_{HC} - t_{LE}$	^oC	47.9	49.6	52.6
28	R	$1/2 \ (t_{WE} + t_{LE})$	^oC	49.2	50.1	46.7
29	V	Density of inflowing air γ	kg/m^3	1.155	1.155	1.170
30						
31	V	Air velocity before radiator W_f	m/s	38.9	32.75	26.85

TABLE I (Cont.)
Experimental and Calculated Data

No.	1	2	3	7	8	9
32	V	Air velocity behind radiator W_f	m/s	16.1	13.6	10.8
33	V	Mean air velocity in the tubes W_V	m/s	23.4	19.8	15.75
34	R	Mean air velocity in the tubes, calculated from eq. 3, 3a and 3b W_R	m/s	21.6	18.1	14.5
35	R	$100 \frac{W_R}{W_V}$	-	92.3	91.4	92.1
36	R	Coefficient of heat transmission for air, inner surface α_{L1}	kg-cal/m²/h/gr	95.4	83.0	70.0
37	R	Reduction factor for unwet inner surface θ	-	0.992	0.993	0.994
38	R	Area of reduced inner surface	m²	19.90	19.92	19.96
39	R	Coefficient of heat transmission, jacket surface α_{L2}	kg-cal/m²/h/gr	106.5	93.0	80.0
40	R	Coefficient of heat transmission, for broad strips of frontal surface α_{L3}	kg-cal/m²/h/gr	287	260	240
41	R	Coefficient of heat transmission, for narrow strips of frontal surface α_{L4}	kg-cal/m²/h/gr	418	390	360
42	R	Mean coefficient of heat transmission for air $\alpha_L = \frac{\Sigma \alpha F}{\Sigma F}$	kg-cal/m²/h/gr	96.2	83.7	70.8
43	R	Mean wall temperature t_i	°C	67.3	68.8	67.9

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	7	8	9
44	R	Mean temperature of marginal water layer $\frac{t_i+t_W}{2}$	°C	-	-	-
45	R	Mean coefficient of heat transmission for water from eq. 25 and 29 α_H	kg-cal/m²/h/gr	767	769	804
46	V-R	Ditto according to eq. 27 from experiments α_{WV}	kg-cal/m²/h/gr	588	578	426
47	R	Mean coefficient of heat transmission k	kg-cal/m²/h/gr	77.3	69.5	60.0
48	R	k F	-	1592	1430	1233
49	R	Water equivalent of the water M	kg/h	3850	3820	4180
50	R	Density of air at $\frac{1}{2}(t_{LE}+t_{WE})$	kg/m³	1.071	1.070	1.075
51	R	Mean density of air at $\frac{1}{2}(t_{LE}+t_{LA})$	kg/m³	1.113	1.105	1.115
52	V	Velocity of water W_W	mm/s	36.1	34.8	38.1
53	R	Water equivalent of the air m	kg/h	2023	1798	1354
54	R	$\frac{kF}{M}$		0.414	0.374	0.395
55	R	$\frac{kF}{m}$		0.787	0.795	0.912
56	R	K_n		0.259	0.236	0.190
57	R	K_K		0.256	0.234	0.188

TABLE I.
Experimental and Calculated Data.

No.	1	2	3	7	8	9
58	R	Total transmitted heat, calculated Q_K	kg-cal/h	47300	44300	41400
59	V	Total transmitted heat, by experiment Q_V	kg-cal/h	45700	42450	38000
60	R	$100 \frac{Q_K}{Q_V}$		103.5	104.5	109.0

V stands for "Versuch" (experiment)

R " " "Rechnung" (calculation)

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	10	11	12	13
1	V	Source	-	Von Doblhoff, Investigation of automobile radiators			
2	V	Experimental number of the source	-	72	73	74	83
3	V	Source and type of radiator	-	Dr. Zimmermann in Ludwigshafen			
4	V	Outside dimensions of radiator h × b × l	mm	475 × 494 × 100 (with flanges 500×500×100)			
5	V	Frontal area b × h	m²	0.2347			
6	V	Number, cross-section and length of tubes	mm	(4046 + 68/2) × × 7.6/7/7.6 × 100			
7	V	Thickness of wall	mm	Estimated 0.15			
8	R	Equivalent diameter d	mm	7.6/7/7.6			
9	V	Total free cross-section of air intake F_e	m²	0.185			
10	V	Total free cross-section of inner air passages F_i	m²	0.157			
11	V	Total free cross-section of air outlet F_a	m²	0.185			
12	R	Air permeability at intake $\varphi_e = \frac{F_e}{F_{st}}$	-	0.788			
13	R	Air permeability within $\varphi_i = \frac{F_i}{F_{st}}$	-	0.669			
14	R	Air permeability at outlet $\varphi_a = \frac{F_a}{F_{st}}$	-	0.788			

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	10	11	12	13
15	V	Inner surface of tubes in contact with water	m^2		9.073		
16	V	Inner surface of tubes, not in contact with water	m^2		-		
17	V	Surface of jacket	m^2		0.095		
18	V	Broad strips of frontal surface - width:area	mm/m^2		0.092		
19	V	Narrow strips of frontal surface - width:area	mm/m^2		0.092		
20	V	Total cooling surface (not reduced)	m^2		9.26		
21	V	Free cross-sections of water passages (maximum)	mm		492 x 86		
22	R	Equivalent diameter of water passages d_w	mm		-		
23							
24	V	Barometer reading	mm Hg	Assumed 760			
25	V	Temperature of inflowing air t_{LE}	°C	19.3	21.9	19.3	20.2
26	V	Temperature of inflowing water t_{WE}	°C	51.3	87.3	92.1	92.5
27	V	Difference in inflow temperature $t_{HC} - t_{LE}$	°C	32.0	65.4	72.8	72.3
28	R	$\frac{1}{2}\,(t_{WE} + t_{LE})$	°C	35.3	54.6	55.7	56.3
29	V	Density of inflowing air γ	kg/m^3	1.205 (assumed at 760 mm Hg and 20°C)			
30							
31	V	Air velocity before radiator W_f	m/s	17.0	17.0	17.0	13.0

TABLE I (Cont.)
Experimental and Calculated Data

No.	1	2	3	10	11	12	13
32	V	Air velocity behind radiator W_f	m/s	10.2	10.2	10.2	7.6
33	V	Mean air velocity in the tubes W_V	m/s	15.2	15.2	15.2	11.4
34	R	Mean air velocity in the tubes, calculated from eq. 3, 3a, and 3b W_R	m/s	15.0	15.0	15.0	11.4
35	R	$100 \cdot \frac{W_R}{W_V}$	-	98.9	98.8	98.8	100.0
36	R	Coefficient of heat transmission for air, inner surface α_{L1}	kg-cal/m²/h/gr	76.1	74.0	73.4	59.4
37	R	Reduction factor for unwet inner surface θ	-	1.0	1.0	1.0	1.0
38	R	Area of reduced inner surface	m²	9.073	9.073	9.073	9.073
39	R	Coefficient of heat transmission, jacket surface α_{L2}	kg-cal/m²/h/gr	56.0	56.0	56.0	41.0
40	R	Coefficient of heat transmission, for broad strips of frontal surface α_{L3}	kg-cal/m²/h/gr	76.6	74.0	74.0	59.4
41	R	Coefficient of heat transmission for narrow strips of frontal surface α_{L4}	kg-cal/m²/h/gr	76.6	74.0	74.0	59.4
42	R	Mean coefficient of heat transmission for air $\alpha_L = \frac{\Sigma \alpha F}{\Sigma F}$	kg-cal/m²/h/gr	76.8	73.6	73.1	59.0
43	R	Mean wall temperature t_i	°C	38.3	54.0	74.2	71.3

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	10	11	12	13
44	R	Mean temperature of marginal water layer $\frac{t_i+t_W}{2}$	°C	40.7	58.3	79.1	75.6
45	R	Mean coefficient of heat transmission for water from eq. 25 and 29 α_H	kg-cal/m²/h/gr	215	213	334	311
46	V-R	Ditto according to eq. 27 from experiments α_{WV}	kg-cal/m²/h/gr	259	213	367	322
47	R	Mean coefficient of heat transmission k	kg-cal/m²/h/gr	56.6	56.3	60.5	49.6
48	R	k F	-	524	522	560	459
49	R	Water equivalent of the water M	kg/h	709	400	2000	1500
50	R	Density of air at $\frac{1}{2}(t_{LE}+t_{WE})$	kg/m³	1.145	1.086	1.070	1.075
51	R	Mean density of air at $\frac{1}{2}(t_{LE}+t_{LA})$	kg/m³	1.162	1.136	1.090	1.180
52	V	Velocity of water W_W	mm/s	4.66	2.63	13.15	9.85
53	R	Water equivalent of the air m	kg/h	2390	2340	2245	1860
54	R	$\frac{kF}{M}$		0.740	1.305	0.280	0.306
55	R	$\frac{kF}{m}$		0.220	0.223	0.250	0.246
56	R	K_n		0.500	0.740	0.221	0.240
57	R	K_K		0.483	0.687	0.220	0.238

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	10	11	12	13
58	R	Total transmitted heat, calculated Q_K	kg-cal/h	10980	17980	32000	25800
59	V	Total transmitted heat, by experiment Q_V	kg-cal/h	11500	18800	32400	25600
60	R	$100 \frac{Q_K}{Q_V}$		95.4	95.5	98.8	100.8

V stands for "Versuch" (experiment)

R " " "Rechnung" (calculation)

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	14	15	16	17
1	V	Source	-	Von Doblhoff, Investigation of automobile radiators			
2	V	Experimental number of the source	-	39	47	52	56
3	V	Source and type of radiator	-	Dr. Zimmermann in Ludwigshafen			
4	V	Outside dimensions of radiator $h \times b \times l$	mm	475 x 494 x 100(with flanges 500×500×100)			
5	V	Frontal area $b \times h$	m²	0.2347			
6	V	Number, cross-section and length of tubes	mm	(4046 + 68/2) x x 7.6/7/7.6 x 100			
7	V	Thickness of wall	mm	Estimated 0.15			
8	R	Equivalent diameter d	mm	7.6/7/7.6			
9	V	Total free cross-section of air intake F_e	m²	0.185			
10	V	Total free cross-section of inner air passages F_i	m²	0.157			
11	V	Total free cross-section of air outlet F_a	m²	0.185			
12	R	Air permeability at intake $\varphi_e = \frac{F_e}{F_{st}}$	-	0.788			
13	R	Air permeability within $\varphi_i = \frac{F_i}{F_{st}}$	-	0.669			
14	R	Air permeability at outlet $\varphi_a = \frac{F_a}{F_{st}}$	-	0.788			

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	14	15	16	17
15	V	Inner surface of tubes in contact with water	m^2	9.073			
16	V	Inner surface of tubes not in contact with water	m^2	–			
17	V	Surface of jacket	m^2	0.095			
18	V	Broad strips of frontal surface - width:area	mm/m^2	0.092			
19	V	Narrow strips of frontal surface - width:area	mm/m^2	0.092			
20	V	Total cooling surface (not reduced)	m^2	9.26			
21	V	Free cross-sections of water passages (maximum)	mm	492 x 86			
22	R	Equivalent diameter of water passages d_W	mm	–			
23							
24	V	Barometer reading	mm Hg	Assumed 760			
25	V	Temperature of inflowing air t_{LE}	°C	20.0	19.4	19.8	20.1
26	V	Temperature of inflowing water t_{WE}	°C	63.7	50.8	63.2	94.0
27	V	Difference in inflow temperature $t_{HC} - t_{LE}$	°C	43.7	31.4	43.4	73.9
28	R	$\frac{1}{2}(t_{WE} + t_{LE})$	°C	41.9	35.1	41.5	57.0
29	V	Density of inflowing air γ	kg/m^3	1.205 (Assumed at 760 mm Hg and 20°C)			
30							
31	V	Air velocity before radiator W_f	m/s	13.0	13.0	9.0	9.0

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	14	15	16	17
32	V	Air velocity behind radiator W_f	m/s	7.6	5.0	5.0	3.6
33	V	Mean air velocity in the tubes W_V	m/s	11.4	7.5	7.5	3.9
34	R	Mean air velocity in the tubes, calculated from eq. 3, 3a, and 3b W_R	m/3	11.4	7.7	7.7	4.2
35	R	$100 \frac{W_R}{W_V}$	–	100.0	102.5	102.5	107.5
36	R	Coefficient of heat transmission for air, inner surface α_{L1}	kg-cal/m²/h/gr	60.9	44.8	44.7	26.8
37	R	Reduction factor for unwet inner surface θ	–	1.0	1.0	1.0	1.0
38	R	Area of reduced inner surface	m²	9.073	9.073	9.073	9.073
39	R	Coefficient of heat transmission, jacket surface α_{L2}	kg-cal/m²/h/gr	41.0	34.3	34.3	22.2
40	R	Coefficient of heat transmission, for broad strips of frontal surface α_{L3}	kg-cal/m²/h/gr	60.9	44.8	44.7	26.8
41	R	Coefficient of heat transmission for narrow strips of frontal surface α_{L4}	kg-cal/m²/h/gr	60.9	44.8	44.7	26.8
42	R	Mean coefficient of heat transmission for air $\alpha_L = \frac{\Sigma \alpha F}{\Sigma F}$	kg-cal/m²/h/gr	60.6	44.7	44.6	26.7
43	R	Mean wall temperature t_i	°C	48.8	44.9	45.7	84.6

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	14	15	16	17
44	R	Mean temperature of marginal water layer $\frac{t_i+t_W}{2}$	°C	51.8	46.7	49.0	87.6
45	R	Mean coefficient of heat transmission for water from eq. 25 and 29 α_H	kg-cal/m²/h/gr	233	281	302	350
46	V-R	Ditto according to eq. 27 from experiments α_{WV}	kg-cal/m²/h/gr	262	284	233	-
47	R	Mean coefficient of heat transmission k	kg-cal/m²/h/gr	48.1	38.5	36.6	24.6
48	R	k F	-	445	357	339	228
49	R	Water equivalent of the water M	kg/h	768	2000	400	2000
50	R	Density of air at $\frac{1}{2}(t_{LE}+t_{WE})$	kg/h	1.120	1.145	1.123	1.070
51	R	Mean density of air at $\frac{1}{2}(t_{LE}+t_{LA})$	kg/m³	1.130	1.150	1.130	1.155
52	V.	Velocity of water W_W	mm/s	5.05	13.15	2.63	13.15
53	R	Water equivalent of the air m	kg/h	1753	1210	1185	666
54	R	$\frac{kF}{M}$		0.592	0.178	0.848	0.114
55	R	$\frac{kF}{m}$		0.254	0.295	0.286	0.342
56	R	K_n		0.416	0.144	0.541	0.0928
57	R	K_K		0.408	0.143	0.524	0.0926

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	14	15	16	17
58	R	Total transmitted heat, calculated Q_K	kg-cal/h	13730	8990	9100	13700
59	V	Total transmitted heat, by experiment Q_V	kg-cal/h	13830	9000	9480	14200
60	R	$100 \frac{Q_K}{Q_V}$		99.4	99.9	97.1	96.5

V stands for "Versuch" (experiment)

R " " "Rechnung" (calculation)

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	18	19	20
1	V	Source	-	Von Doblhoff, Investigation of automobile radiators		
2	V	Experimental number of the source	-	65	79	76
3	V	Source and type of radiator	-	Dr. Zimmermann in Ludwigshafen		
4	V	Outside dimensions of radiator $h \times b \times l$	mm	475 × 494 × 100 (with flanges 500×500×100)		
5	V	Frontal area $b \times h$	m²	0.2347		
6	V	Number, cross-section and length of tubes	mm	(4046 + 68/2) × × 7.6/7/7.6 × 100		
7	V	Thickness of wall	mm	Estimated 0.15		
8	R	Equivalent diameter d	mm	7.6/7/7.6		
9	V	Total free cross-section of air intake F_e	m²	0.185		
10	V	Total free cross-section of inner air passages F_i	m²	0.157		
11	V	Total free cross-section of air outlet F_a	m²	0.185		
12	R	Air permeability at intake $\varphi_e = \frac{F_e}{F_{st}}$	-	0.788		
13	R	Air permeability within $\varphi_i = \frac{F_i}{F_{st}}$	-	0.669		
14	R	Air permeability at outlet $\varphi_a = \frac{F_a}{F_{st}}$	-	0.788		

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	18	19	20
15	V	Inner surface of tubes in contact with water	m^2		9.073	
16	V	Inner surface of tubes not in contact with water	m^2		-	
17	V	Surface of jacket	m^2		0.095	
18	V	Broad strips of frontal surface - width:area	mm/m^2		0.092	
19	V	Narrow strips of frontal surface - width:area	mm/m^2		0.092	
20	V	Total cooling surface (not reduced)	m^2		9.26	
21	V	Free cross-sections of water passages (maximum)	mm		492 x 86	
22	R	Equivalent diameter of water passages d_W	mm		-	
23						
24	V	Barometer reading	mm Hg		Assumed 760	
25	V	Temperature of inflowing air t_{LE}	°C	20.5	20.4	20.0
26	V	Temperature of inflowing water t_{WE}	°C	86.9	95.5	60.3
27	V	Difference in inflow temperature $t_{HC} - t_{LE}$	°C	66.4	75.1	40.3
28	R	$\frac{1}{2}(t_{WE} - t_{LE})$	°C	53.7	57.9	40.2
29	V	Density of inflowing air γ	kg/m^3		1.205	
30					(Assumed at 760 mm Hg and 20°C)	
31	V	Air velocity before radiator W_f	m/s	5.0	2.5	2.5

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	18	19	20
32	V	Air velocity behind radiator W_f	m/s	2.6	1.12	1.21
33	V	Mean air velocity in the tubes W_V	m/s	3.9	1.68	1.81
34	R	Mean air velocity in the tubes, calculated from eq. 3, 3a and 3b W_R	m/s	4.2	2.0	2.0
35	R	$100 \frac{W_R}{W_V}$	–	107.5	119.0	110.0
36	R	Coefficient of heat transmission for air, inner surface α_{L1}	kg-cal/m²/h/gr	26.8	15.3	15.3
37	R	Reduction factor for unwet inner surface θ	m²	1.0	1.0	1.0
38	R	Area of reduced inner surface	m²	9.073	9.073	9.073
39	R	Coefficient of heat transmission, jacket surface α_{L2}	kg-cal/m²/h/gr	22.2	13.0	13.0
40	R	Coefficient of heat transmission for broad strips of frontal surface α_{L3}	kg-cal/m²/h/gr	26.8	15.3	15.3
41	R	Coefficient of heat transmission for narrow strips of frontal surface α_{L4}	kg-cal/m²/h/gr	26.8	15.3	15.3
42	R	Mean coefficient of heat transmission for air $\alpha_L = \frac{\Sigma \alpha F}{\Sigma F}$	kg-cal/m²/h/gr	26.7	15.3	15.3
43	R	Mean wall temperature t_i	°C	63.0	90.5	56.0

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	18	19	20
44	R	Mean temperature of marginal water layer $\frac{t_i + t_W}{2}$	°C	65.3	91.8	57.0
45	R	Mean coefficient of heat transmission for water from eq. 25 and 29 α_H	kg-cal/m²/h/gr	205	354	280
46	V-R	Ditto according to eq. 27 from experiments α_{WV}	kg-cal/m²/h/gr	–	–	–
47	R	Mean coefficient of heat transmission k	kg-cal/m²/h/gr	23.5	14.6	14.5
48	R	k F	–	217	134.5	134.0
49	R	Water equivalent of the water M	kg/h	250	2000	1500
50	R	Density of air at $\frac{1}{2}(t_{LE} + t_{WE})$	kg/h	1.080	1.068	1.130
51	R	Mean density of air at $\frac{1}{2}(t_{LE} + t_{LA})$	kg/m³	1.173	1.135	1.170
52	V	Velocity of water W_W	mm/s	1.64	13.15	9.43
53	R	Water equivalent of the air m	kg/h	676	312	322
54	R	$\frac{kF}{M}$		0.868	0.0673	0.0892
55	R	$\frac{kF}{m}$		0.321	0.431	0.416
56	R	K_n		0.544	0.054	0.0715
57	R	K_K		0.526	0.054	0.0712

TABLE I (Cont.)
Experimental and Calculated Data.

No.	1	2	3	18	19	20
58	R	Total transmitted heat, calculated Q_K	kg-cal/h	8750	8110	4320
59	V	Total transmitted heat, by experiment Q_V	kg-cal/h	9500	9600	5400
60	R	$100 \frac{Q_K}{Q_V}$		92.1	84.5	80.2

V stands for "Versuch" (experiment)

R " " "Rechnung" (calculation)

c) Table I.- Here a large number of foreign experiments is compared with the values calculated directly from the dimensions, the velocity of the air in front of the radiator, the quantity of cooling water and the temperature of the inflowing water, whereby the applicability of the process of calculation is demonstrated throughout.

The values taken directly from the experimental results or obtained from them by simple calculations are denoted by "V" and the values obtained from the above-evolved process of calculation are denoted by "R."

The calculated velocity of the air in the tubes, which has the greatest effect on the heat transmission, agrees very well with the experimental values up to 4 m/s (corresponding to about 5 m/s in front of the radiator according to Doblhoff). Below this limit, which corresponds to the critical velocity in the 7 mm tubes, the calculation also gives fairly accurate values for the Zimmermann radiator, as is indeed to be expected from the shortness of the distance for the transition from the vortical to the laminar flow. Still the formulas should be applied only with caution to very deep radiators (over 150 mm), when W is less than 4.

Probably the measured values of these small velocities are affected by the errors under consideration, wherefore Doblhoff did not take up, in his report, the heat proceeding from the temperature and velocity of the air in the experiments with 2.5 m/s.

The calculated values of the transmitted heat agree very well, almost all the way through, with the experimental values. They must be somewhat smaller than the values determined from the quantity and temperature of the cooling water, and indeed by an amount (in % of the transmitted heat), increasing with decreasing air velocity, which represents the radiated portion. Even here the calculated results agree, in general, with the experimental results.

Greater differences occur, in comparison with Doblhoff's experiments at a speed of 2.5 m/s, which is quite comprehensible from what has been said and with the Daimler radiator of 300 mm depth. Here great differences are also manifested between the directly calculated coefficients of heat transmission for water and those obtained from experiments with the help of the coefficient of heat transmission for air, though they agree very well ordinarily. I ascribe these differences to the unequal water velocities in the individual passages, due to the great depth of the radiator. No positive explanation, however, is now possible, but even here the greatest difference between the calculated and experimental results is only 9%.

For rough calculations, it will suffice to add the frontal area and possibly also the jacket surface to the inside surface of the tubes and calculate throughout with the coefficient of heat transmission for air in the tube. Even the effect of intermediate cooling, as likewise the radiation, can be disre-

garded without important error, especially as these partially offset one another. On the other hand, it seems entirely inadmissible to put the coefficient of heat transmission for water so high, as was done throughout (after having been begun by Doblhoff) in the literature on the subject, without considering the velocity of the water, since the error can be very large, especially with high air velocities and low water velocities.

EFFECT OF DIMENSIONS AND WORKING CONDITIONS ON THE HEAT EXCHANGE.

Since it is not easy, with the aid of the equations alone, to determine how the cooling process is affected by the shape of the cross-section and the dimensions of the tubes and of the radiator core, the velocity of the air, the quantity of the water, the entrance temperatures of the air and water and the density of the air, these influences must be systematically and thoroughly investigated, but with limitation of the heat exchange to the walls of the tubes, since the consideration of the frontal and lateral surfaces of the shell would make the determination difficult, while their share in the heat transmission, as was explained above, would be relatively small in comparison with the inner surfaces of the tubes and could be easily allowed for subsequently. The small amount of heat lost by radiation will also be here disregarded.

1. Cross-sectional shape of the tubes.- According to the principal equations (4a), (16), (17), (18a) and (25), the heat exchange (with constant frontal and cooling surfaces, depth of core, and velocity of water and air) depends only on the equivalent diameter of the tubes

$$d = \frac{4f}{u} \tag{1}$$

and on the air permeability φ. If we previously put $\varphi = 1$, which presupposes very thin walls to the tubes and very small water cross-sections and a frontal area completely filled by the tubular cross-sections, it then follows that radiators with like frontal areas and depth of core and having the same ratio of frontal area to cooling surface are equivalent.

For n tubes, the frontal area $F_{st} = nf$ and the cooling area in contact with the air $F = nul$ follows from equation (1)

$$d = \frac{4\ F_{st}/n}{F/(n\,l)} = 4\,l\ F_{st}/F \tag{1a}$$

in which l remains unchanged, according to the assumption.

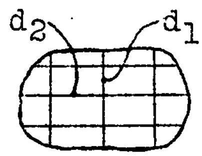
TABLE III.

Comparison of Thin-Walled Tubes of Various Sizes and Cross-Sectional Shapes with Square Tubes 6 × 6 mm².

Frontal surface of block $F_{st} = 1\ m^2$

Length of tubes $l = 100$ mm

Cooling surface $F = 66.77\ m^2$



d_1/d_2			0.6	1.0	1.5	2.0
d_1		mm	12.95	10.2	12.75	14.28
d_2		mm	19.9	10.2	8.5	7.64
Equivalent diameter	d	mm	6.0	6.0	6.0	6.0
Number of tubes per m²	n		15200	21750	19600	17450
Cross-sectional area of one tube	f_{st}	mm²	65.8	46.2	51.0	57.0
Cooling surface of one tube	f	mm²	4380	3060	3400	3820
Air permeability	$\varphi_i = \frac{n\ f_{st}}{F_{st}}$		1.0	1.0	1.0	1.0

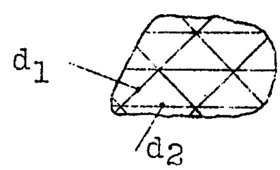
TABLE III (Cont.)

Comparison of Thin-Walled Tubes of Various Sizes and Cross-Sectional Shapes with Square Tubes 6 × 6 mm².

Length of tubes l = 100 mm

Frontal surface of block F_{st} = 1 m²

Cooling surface F = 66.77 m²



d_1/d_2			0.2	0.5	1.0	1.5	2.0
d_1		mm	6.14	6.66	8.49	10.8	13.32
d_2		mm	30.7	13.32	8.49	7.20	6.62
Equivalent diameter	d	mm	6.0	6.0	6.0	6.0	6.0
Number of tubes per m²	n		10650	22600	27800	25700	22600
Cross-sectional area of one tube	f_{st}	mm²	94.0	44.4	36.0	38.9	44.4
Cooling surface of one tube	f	mm²	6200	2940	2400	2590	29.5
Air permeability	$\varphi_i = \frac{n\,f_{st}}{F_{st}}$		1.0	1.0	1.0	1.0	1.0

TABLE III (Cont.)
Comparison of Thin-Walled Tubes of Various Sizes and Cross-Sectional Shapes with Square Tubes 6 × 6 mm².

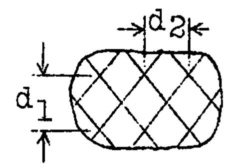
			Cooling surface $F = 66.77\ m^2$				
Frontal surface of block $F_{st} = 1\ m^2$ Length of tubes l = 100 mm							
d_1/d_2			0.2	0.5	1.0	1.5	2.0
d_1		mm	3.6	4.5	6.0	7.5	9.0
d_2		mm	18.0	9.0	6.0	5.0	4.5
Equivalent diameter	d	mm	6.0	6.0	6.0	6.0	6.0
Number of tubes per m²	n		15400	24700	27777	26700	24700
Cross-sectional area of one tube	f_{st}	mm²	64.8	40.5	36.0	37.5	40.5
Cooling surface of one tube	f	mm²	4320	2700	2400	2500	2700
Air permeability $\varphi_i = \frac{n\ f_{st}}{F_{st}}$			1.0	1.0	1.0	1.0	1.0

TABLE III (Cont.)

Comparison of Thin-Walled Tubes of Various Sizes and Cross-Sectional Shapes with Square Tubes 6 × 6 mm².

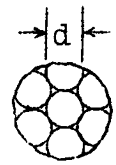
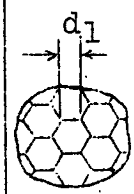
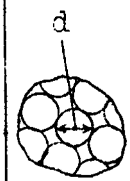
Frontal surface of block $F_{st} = 1\ m^2$ Length of tubes $l = 100$ mm Cooling surface $F = 66.77\ m^2$					
d_1/d_2			–	–	–
d_1		mm	3.464	4.72	5.44
d_2		mm	–	–	–
Equivalent diameter	d	mm	6.0	4.72	5.44
Number of tubes per m²	n		32100	45000	38800
Cross-sectional area of one tube	f_{st}	mm²	31.2	17.5	23.24
Cooling surface of one tube	f	mm²	2028	1483	1709
Air permeability	$\varphi_i = \frac{n\ f_{st}}{F_{st}}$		1.0	0.7854	0.905

Table III contains, for tubes of various cross-sectional shapes which satisfy equation (1a) and are equivalent to square tubes, 6 mm square, the widths of the sides, the cross-sectional areas, the perimeters and number of tubes per 1 m² of frontal area. For comparison, the same data were calculated for equivalent circular cross-sections with quadratic and rhombic distribution of the central points. The greatest air permeability

(for very thin walls and very narrow water passages), however, is here only

$\varphi = 0.7854$ with quadratic distribution of the central points,

$\varphi = 0.905$ " rhombic " " " " "

It is therefore considerably less favorable than with surface-filling, straight-sided cross-sections having the quantity of material which is characterized by the area of the cooling surface for 100 mm depth of core for all cross-sections.

$$F = n\ u\ l = 66.67\ m^2$$

Since, for the regular cross-sectional shapes of the tubes (triangle, square, hexagon, circle), the greatest number of tubes occurs and the work and consumption of solder generally increase with the number of tubes, these shapes also necessitate the greatest production cost of the core for an equal cooling effect.

In order to show the effect of the wall thickness $\alpha/2$ (ordinarily 0.1 - 0.2 mm) and of the width of the water passages α_W (usually 1 - 2.5 mm), Fig. 7 represents the transmitted heat for rectangular cross-sections, vertical water passage and a constant ratio F_{st}/F, with the ratio of the sides $d_1/d_2 = 0.2 - 3$ and in comparison with square tubes of 6 x 6 mm² cross-section at an air velocity $W_f = 10$ m/s.

$F_{st} = 1\ m^2$, $F = 44.486\ m^2$, $\alpha/2 = 0.15$ mm, $\alpha_W = 2$ mm (Fig. 5), soldered joint $g = 0.2$ mm and the depth of the solder

on the frontal surface (5 mm) here remain unchanged, so that the water cross-sections are 90 mm long. The water equivalent of the passing water $M = 10000$ kg/h; the temperature of the inflowing water $t_{WE} = 60^{\circ}C$; the temperature of the inflowing air $t_{LE} = 20^{\circ}C$; the density of the inflowing air $\gamma_E = 1.2$ kg/m³. In equations (4b) and (18a), $\gamma_m = 1.122$ was put as the mean density of the air, corresponding to a mean temperature of 40°C. The water equivalent of the passing air m was calculated for equations (13) and (16) from its density at the true mean exit temperature.

The effect of the intermediately cooled coils was determined by multiplication of the intermediately cooled surfaces in contact with the air F_{ml} by the correction factor θ (Fig. 6) with $\lambda = 100$ kg-cal/m/h/°C. The effect of the cross current was calculated according to equations (16) and (16a) and the difference between the cooling surfaces in contact with water and air was calculated according to equation (17).

As shown by Fig. 7, even here the law is approximately correct that radiators having the same frontal area and depth of core with the same ratio of frontal area to cooling surface are equivalent, but a maximum value for Q is nevertheless to be recognized, and indeed at a ratio d_1/d_2 = about 0.6. This value of Q is about 1.5% greater than for a core composed of square tubes.

With greater values of d_1/d_2, the diminished air permea-

bility has a detrimental effect, while at smaller values of d_1/d_2, the diminished utilization of the intermediate cooled surfaces has a detrimental effect. Even here, however, the ratio Q/n, for approximately equal widths of the sides of the tubes, has a minimum value, for which there corresponds a maximum value in work and solder consumption.

The improvement in the cooling effect through the use of rectangular tubes, whose narrower sides are parallel to the direction of the water flow, with a ratio of the sides $d_1/d_2 = 0.5 - 0.8$, is nevertheless worth noting, since it can be attained without any greater outlay in work or material.

2. Size and length of the tubes.- Figs. 8-10 show, for tubes of 2 - 15 mm square (inside) and 25 - 500 mm long (depth of core), with vertical water passages and with an air velocity of 10 m/s in front of the radiator, the air velocities in the tubes, the coefficients of heat transmission for air and water and the quantities of heat transmitted. Thereby there remain unchanged $F_{st} = 1 \times 1\ m^2$, $\alpha/2 = 0.15$ mm (Figs. 5 and 11), $\alpha_W = 2$ mm, $g = 0.2$ mm, $M = 10000$ kg/h, $t_{WE} = 60°C$, $t_{LE} = 20$ C, $\gamma_{LE} = 1.2\ kg/m^3$.

The soldering of the frontal surfaces was assumed to be 5 mm deep on both sides, so that the water compartments had a depth of $l_1 = l - 10$.

The number of tubes, air permeability, water-cooled and

air-cooled cooling surfaces are shown in Fig. 11 for a core 100 mm deep. The effect of the cross current and of the intermediate wall cooling and the difference between the cooling areas, in contact with the water and with the air, were considered the same as in the first part of this report, and the air density $\gamma_m = 1.122$, corresponding to a mean temperature of 40°C was introduced into equations (4b) and (18a). The density at the mean exit temperature of the air was used for calculating the water equivalent m.

The tables and diagrams demonstrate an important law, namely, that for a given frontal area, air velocity and water flow, the cooling effect can be increased only to a certain definite maximum by increasing the cooling area, either by lengthening the tubes or increasing their number with a simultaneous corresponding reduction in their size. The cooling effect is further diminished by any further increase in the cooling area, demonstrating that such increase is without value. Since, in the vicinity of the maximum value of the cooling effect, its increase or decrease is very slow, any lengthening of the tubes or increasing their number is uneconomical, even in a large region below this maximum of the $Q-d-l$ area.

With increasing length of the tubes and unchanged number of tubes (and consequently unchanged cross-section of each tube), the maximum cooling effect and the flat course of the $Q-l$ curve occur in their vicinity and indeed the maximum cooling

values for longer tubes also occur in larger cross-sections.

A similar fact also holds true, with an increasing number of tubes (decreasing cross-section) and unchanged length, for the Q-d curve. The maximum values of the cooling effect also occur with greater tube lengths and diameters.*

The maximum cooling effect, even for greater tube lengths and numbers (smaller diameters), is produced by greater air velocities, though the difference is relatively small.

In order to make the relations clear, Fig. 12 shows the effect of the size and length of the tubes at an air velocity of 10 m/s in front of the radiator.

These results agree very well with the experimental results of the Vienna Engine Works, which show but very little greater cooling effect with airplane radiators of 300 mm depth than with those of 200 mm depth. These results also accord very well with the fact that nearly all automobile radiators, designed on the basis of many years' experience, have tubes with cross-sectional areas of 20-70 mm^2 and cores seldom over 150 mm deep.

<u>3. The frontal shape of the core</u> of equal depth at all points, under like conditions in other respects (especially with

* That maximum values for Q must occur with changeable d and l, can be concluded directly from the fact that, with very small d, the cooling area is large and the air velocity in the tubes is very small, but that, at very large values of d, the air velocity is nearly equal to the velocity of the free air stream, though the cooling area is small. With a core of small depth, the cooling surface is also small, but with a core of great depth, the air velocity in the tubes is small. In extreme cases, the quantity of transmitted heat is therefore small.

unchanged water velocity), has no influence on the cooling effect. This can be demonstrated in the following manner for shallow radiators in which we may assume the mean air temperature in each tube to be the arithmetical mean between the inflow and outflow temperature.*

In a horizontal radiator section (Fig. 13) of the thickness Δ_x, the temperature of the inflowing water is t_{WE}; of the outflowing water, t_{WA}; of the inflowing air, T_{LE}; of the outflowing air t_{LA}; the water equivalent of the water flowing through per hour is M and of the air is

$$\Delta m = m_1 \; y \, \Delta \; x \tag{30}$$

in which y is the width of the section and m_1 the water equivalent of the quantity of air flowing through a unit area per hour. The water enters the radiator itself at the constant temperature T_{WE}.

If f_1 denotes the cooling area corresponding to 1 m² of frontal area, then the cooling area, corresponding to the frontal area of the radiator section, is

$$\Delta \; F = f_1 \; y \; \Delta \; x \tag{31}$$

The following equations hold good for the heat ΔQ transmitted in the section Δ_x and for the heat Q transmitted in

* Without this assumption, the calculation would have had to be similar to that of Nusselt in "Der Wärmeübergang im Kreuzstrom" already referred to. In my opinion, however, the result of the approximate calculation is more lucid.

the portion x of the core

$$\left.\begin{aligned} \Delta Q &= (t_{LA} - T_{LE})\,\Delta m \\ \Delta Q &= k\,\Delta F\left(\frac{t_{WE} + t_{WA}}{2} - \frac{T_{LE} + t_{LA}}{2}\right) \\ Q &= (T_{WE} - t_{WE})\,M \end{aligned}\right\} \tag{32}$$

whence $\frac{\Delta Q}{\Delta m} + \frac{2}{k}\frac{\Delta Q}{\Delta F} + \frac{Q}{M} = T_{WE} + t_{WA} - 2\,T_{LE}$

and, with $t_{WA} = T_{WE} - \frac{Q}{M} - \frac{\Delta Q}{M}$,

we have $$\frac{\Delta Q}{\Delta m} + \frac{2}{k}\frac{\Delta Q}{\Delta F} + \frac{2Q}{M} + \frac{\Delta Q}{M} = 2\,(T_{WE} - T_{LE}) \tag{33}$$

By introducing the values for ΔF and Δm from equations (30) and (31) and disregarding the very small term $\Delta Q/M$, as also by replacing the differences by differentials, which is possible with a fair degree of approximation with the usual relatively large number of tubes, we obtain

$$2\,\frac{Q}{M} + \left(\frac{1}{m_1} + \frac{2}{k\,f_1}\right)\frac{d\,Q}{y dx} - 2\,(T_{WE} - T_{LE}) = 0 \tag{34}$$

and, for the total transmitted heat

$$Q = M\,(T_{WE} - T_{LE})\left(1 - e^{-\frac{\int_0^h y dx}{M\left(\frac{1}{2m_1} - \frac{1}{kf_1}\right)}}\right) \tag{35}$$

The frontal area is represented by $\int_0^h y dx$, regardless of the relation of the width y to x. Hence Q is also independent of the shape of the frontal surface, although dependent on its size. Equation (35) is an approximation between equations (13) and (16).

<u>4. The magnitude of the frontal area</u> generally affects also the quantity and velocity of the water flowing through, but not the coefficient of heat transmission for air. The water equivalent of the passing air is proportional to the frontal area and therefore kF/m is not changed in equation (13) by changing the frontal area. The effect of the water velocity on the coefficient of heat transmission is relatively small and depends also on the ratio of the sides of the radiator tubes. Since, as shown by equation (13), it is further reduced by the simultaneous change in the numerator and denominator, it will be disregarded in this section and the coefficient of heat transmission k will be regarded as constant. Increasing the cooling area can effect an increase, equality or decrease of the water resistance and of the total cross-section of the water passages and hence simultaneously a corresponding change in quantity of water flowing through, so that the following cases must be distinguished.

a) The quantity of water flowing through increases in proportion to the frontal area. Since the water equivalent of the passing air is always proportional to the frontal area (the other structural dimensions remaining unchanged), K_n (or K_k) does not change according to equations (13) and (16) and the transmitted heat is likewise proportional to the quantity of water and hence also to the frontal area (Fig. 14).

b) The quantity of water and of transmitted heat increase

faster than the frontal area.

c) The quantity of water and of transmitted heat increase slower than the frontal area.

A special case is when the quantity of water remains constant. For this case, the effect of changing the frontal area on the cooling action is shown by Fig. 14 on the radiator with tubes having a cross-section 6 x 6 mm² and a length (depth of core) of 100 mm, with an air velocity of 10 m/s and a quantity of water of $M = 10000$ kg/h, whereby the other relations correspond to the radiator mentioned in section 2. The heat transmitted per unit of frontal area decreases, in this case, as the frontal area is increased. If the quantity of water decreases with increasing frontal area, the effect on the transmitted heat is naturally increased. In any case, the character of the QF_{st} curve depends largely on the means employed for keeping the water in motion.

<u>5. The velocity of the air stream in front of the radiator</u> has the greatest effect on the cooling action, as follows, from equations (4a), (16), (17), and (18).

Fig. 15 shows the effect of the air velocity in the case of the radiator with square tubes 6 x 6 mm², 100 mm depth of core, 1 m² frontal area, 10000 kg/h of water and under the other conditions given in section 2. Likewise, Fig. 16 gives the customary practical dimensions for square tubes in the logarithmic scale. It shows that an approximation formula of the

form

$$Q = A\ W_f^{\ r} \tag{36}$$

cannot be established for square tubes with constant r, but that r can, nevertheless, be assumed constant for quite wide velocity ranges.

$$W_f = 2 \quad 20 \quad 50 \text{ m/s}$$

$$r = 0.5 \quad 0.7 \quad 0.85 \text{ (about)}$$

This is facilitated by experiments with engine and radiator combined.

6. The effect of the quantity of water flowing through (as shown by Fig. 17, for the square-tube radiator also taken as an example in the above section at 10 m/s air velocity in front of the radiator) is considerable for small quantities of water, but grows continually less important with increasing quantities of water, as ordinarily used in automobile radiators. With "thermosyphon" radiators using only small quantities of water, we must therefore calculate on a diminished cooling effect.

7. The effect of the air density in front of the radiator is shown by Fig. 18, for the above-described square-tube radiator with unchanged air velocity, quantity of air and temperature of both air and water. The transmitted heat increases nearly in a direct proportion to the increasing density, allowance for which must be made in aircraft radiators (in which, how-

however, the low air temperature partially offsets the effect of the low air pressure) and in automobile radiators in elevated countries.

8. The temperatures of the inflowing air and water, as shown by Figs. 19-20, for square-tube radiators at constant air velocity, air density and quantity of water, have but little effect on the characteristic K_k, so that the transmitted heat under otherwise unchanged conditions is nearly proportional to the differences in the entrance temperatures.

It could also be demonstrated that, with the known laws of fluid flow and heat transmission, the heat abstraction by tubular radiators in an air current above the critical velocity can be very accurately calculated; that, for short-tube radiators, these laws apply approximately even at subcritical velocities; that the radiation is small and that intermediately cooled areas of the practically occurring order of magnitude are almost equivalent to the directly cooled areas.

Although individual problems (that is, heat abstraction by deep radiators at small air velocities and the laws of heat transmission for water) require further elucidation through individual researches and although experimental confirmation of the results appears desirable, the characteristics of tubular radiators may, however, be thus satisfactorily determined, so far as requisite for practical purposes.

A subsequent investigation will have to do with the problem of the best dimensions for a radiator, which will satisfactorily abstract the heat, with the smallest weight and minimum air resistance and at the lowest cost of production.

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

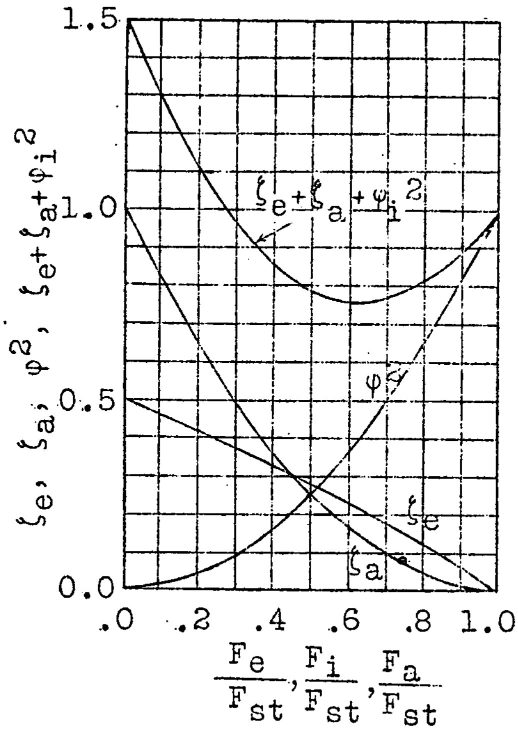


Fig.1

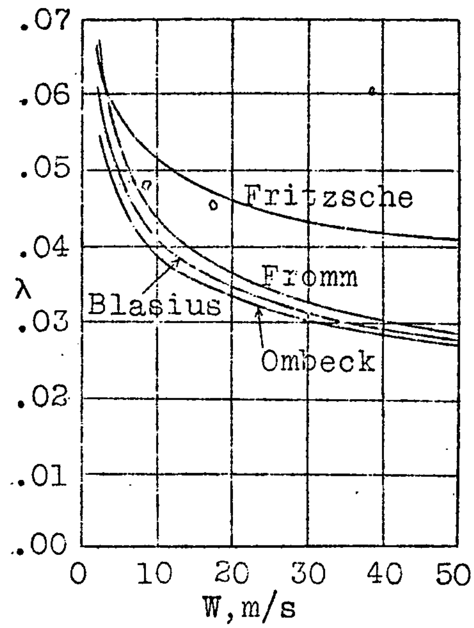


Fig.2

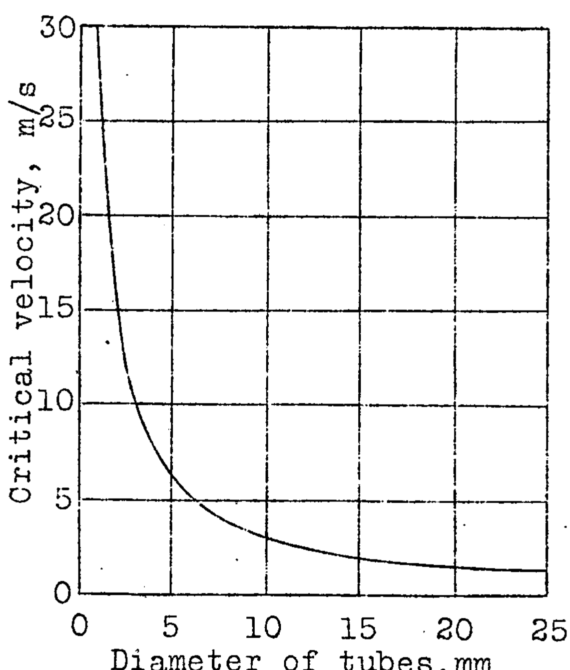


Fig.3

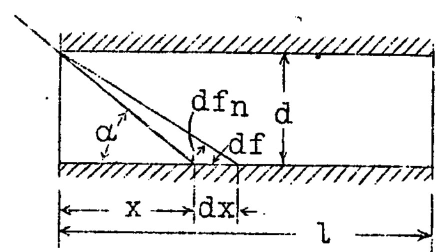


Fig.4

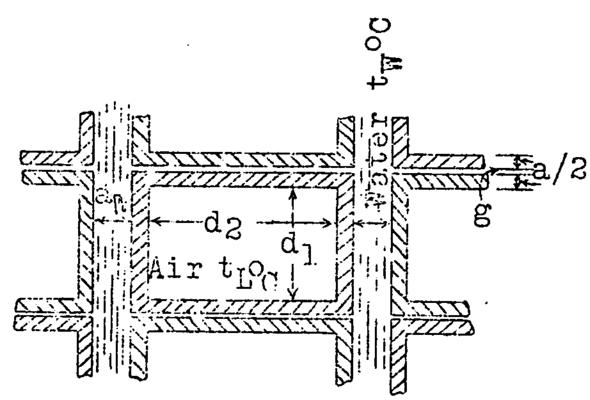


Fig.5

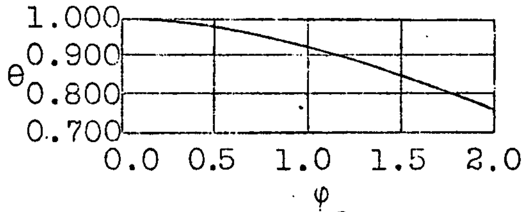


Fig.6

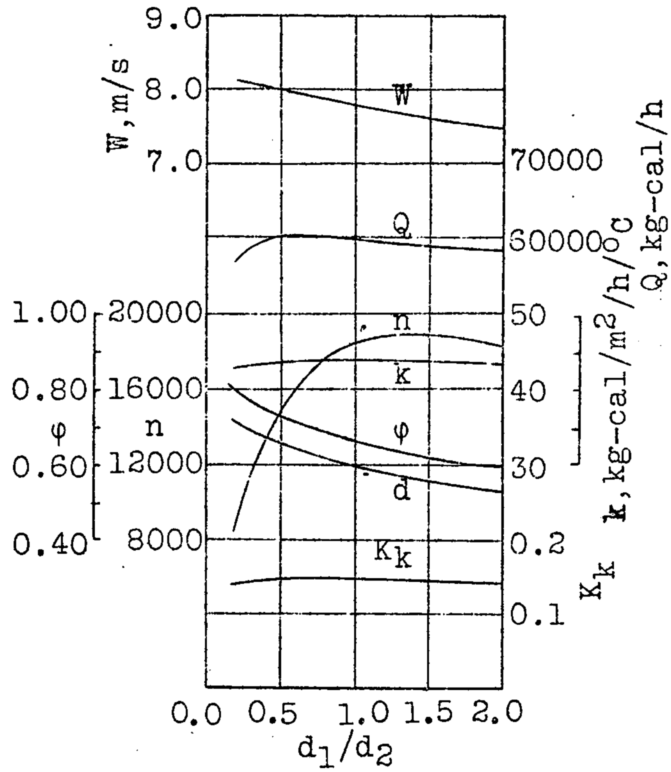


Fig.7

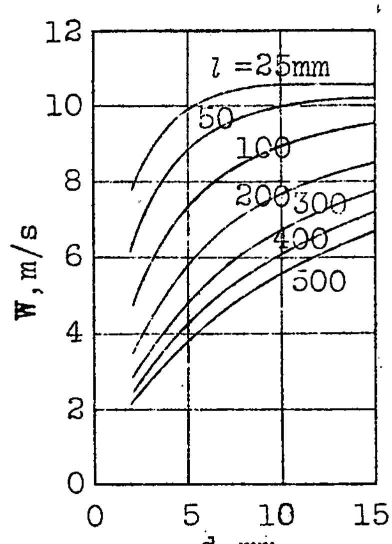


Fig.8

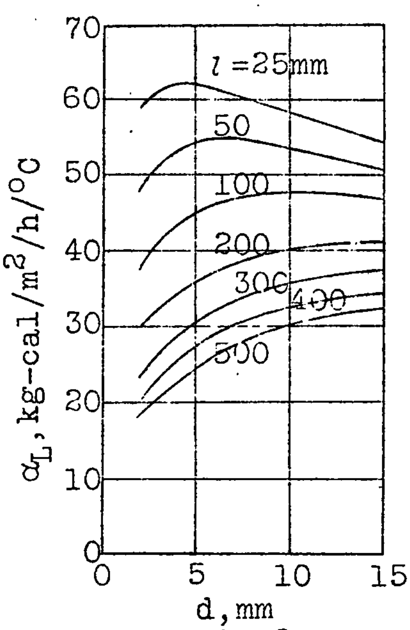


Fig.9

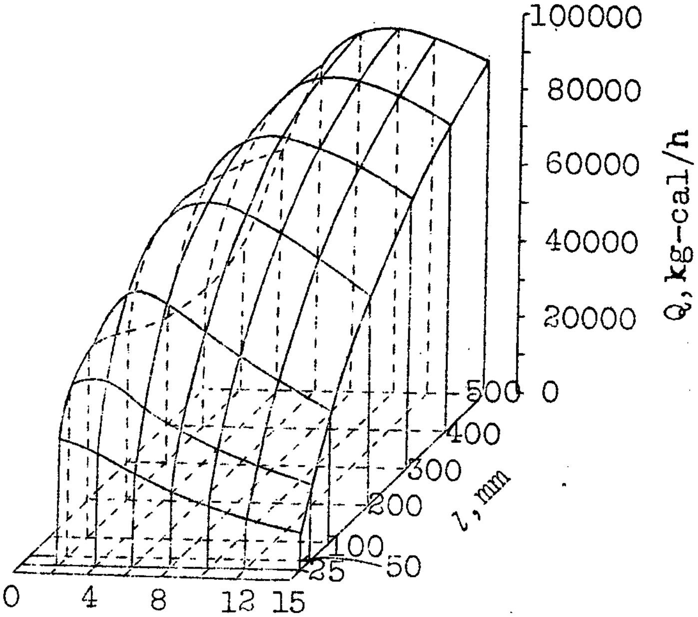


Fig.10

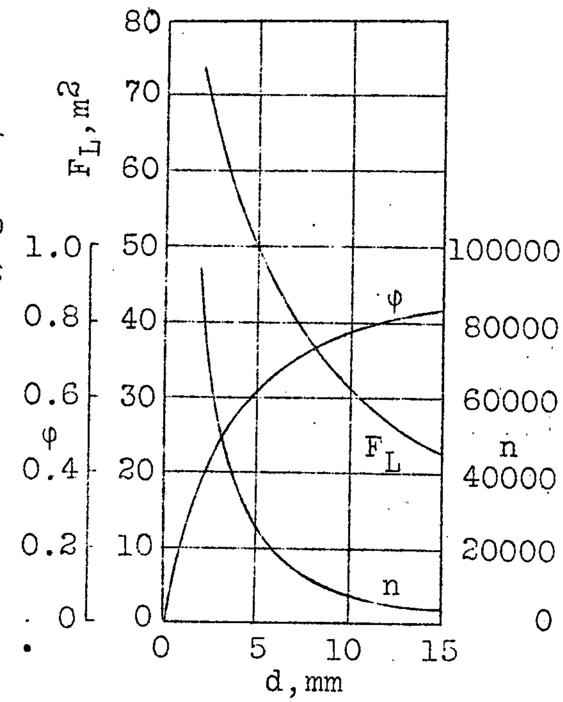


Fig.11

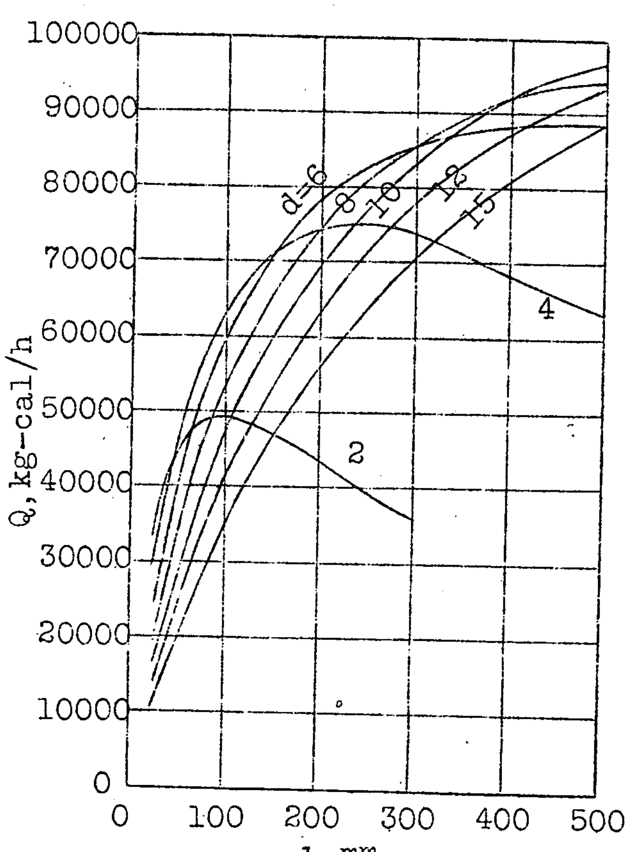


Fig.12

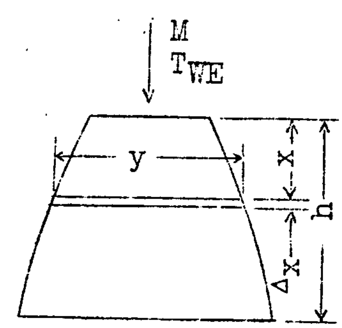


Fig.13

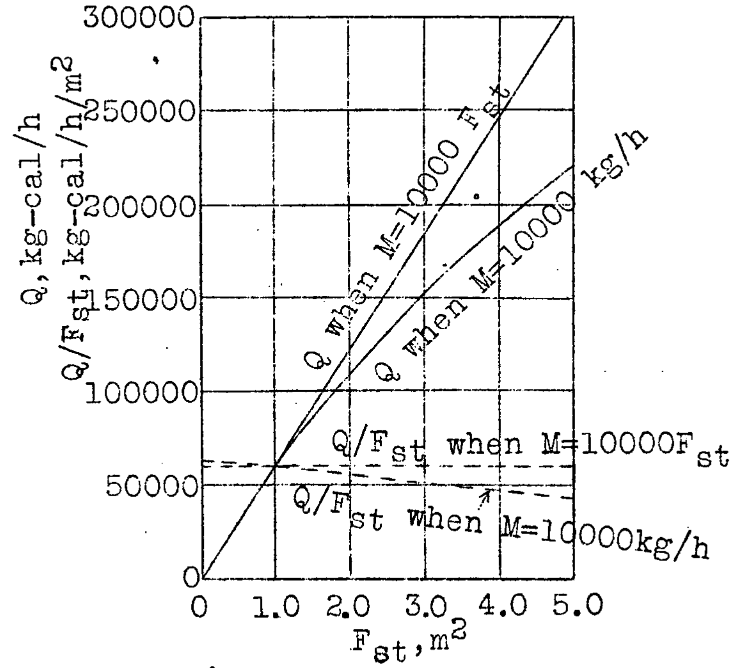


Fig.14

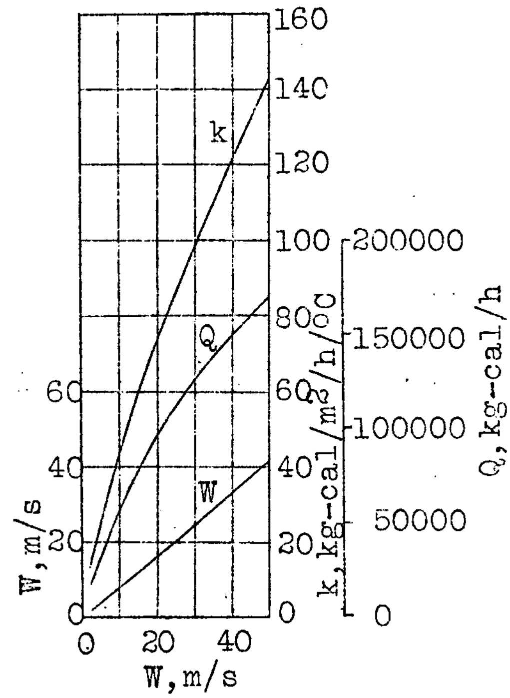


Fig.15

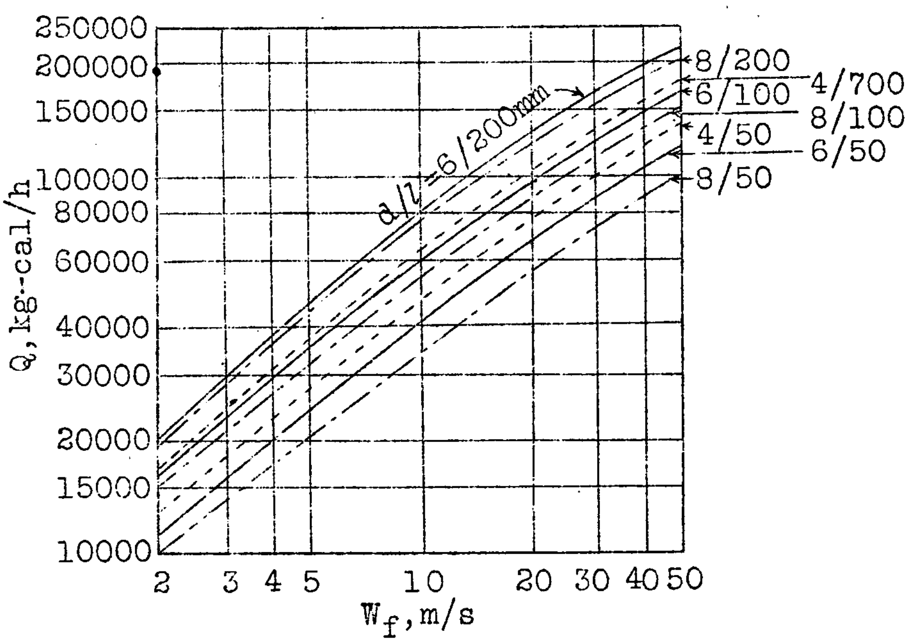


Fig.16

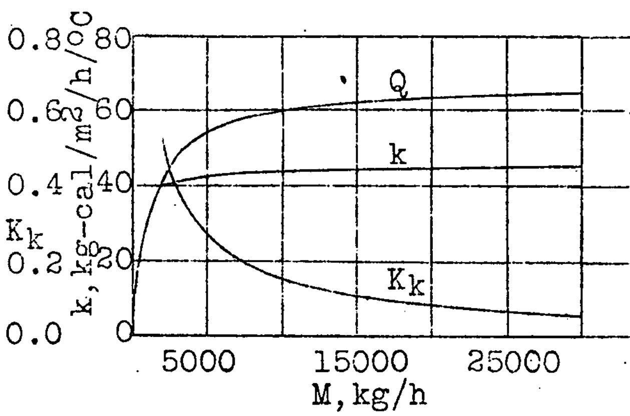


Fig.17

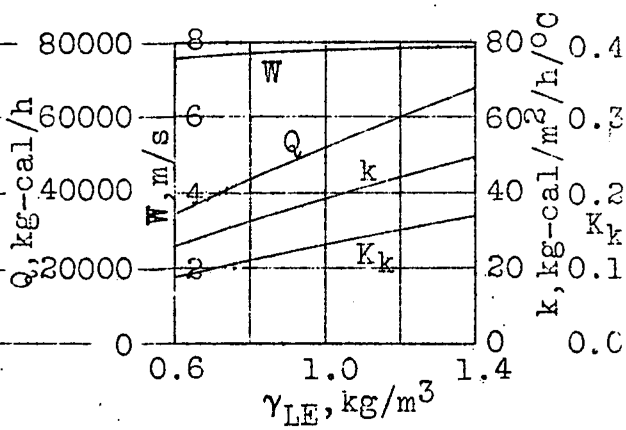


Fig.18

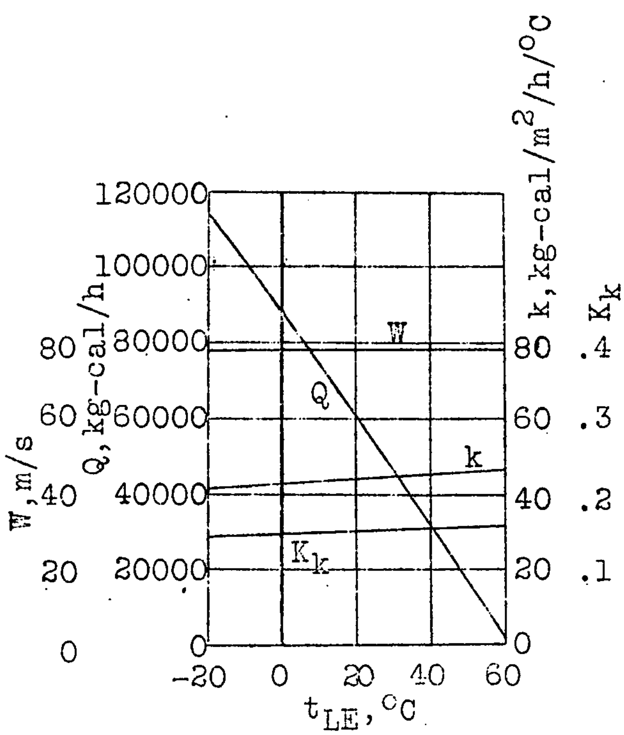


Fig.19

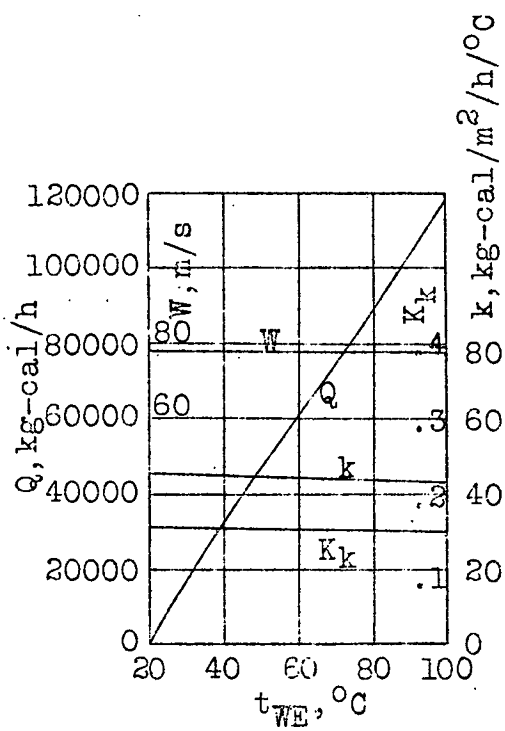


Fig.20